

# CONTEMPORARY MATHEMATICS

172

## Chaotic Numerics

An International Workshop  
on the Approximation and Computation  
of Complicated Dynamical Behavior  
July 12–16, 1993  
Deakin University, Geelong, Australia

Peter E. Kloeden  
Kenneth J. Palmer  
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Chaotic Numerics: An International Workshop on the Approximation and Computation of Complicated Dynamical Behaviour was held at Deakin University, Geelong, Australia, from July 12 to July 16, 1993, with support from the Faculty of Science and Technology of Deakin University and the Australian Mathematical Society.

1991 *Mathematics Subject Classification*. Primary 34A50, 58F13; Secondary 65L99.

Papers in this volume are final versions, which will not be published elsewhere, unless otherwise indicated by a footnote.

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### Library of Congress Cataloging-in-Publication Data

Chaotic numerics: an international workshop on the approximation and computation of complicated dynamical behavior, Deakin University, Geelong, Australia, July 12–16, 1993 / Peter E. Kloeden, Kenneth J. Palmer, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132; v. 172)

Includes bibliographical references.

ISBN 0-8218-5184-5

1. Differentiable dynamical systems—Congresses. 2. Numerical analysis—Congresses. 3. Chaotic behavior in systems—Congresses. I. Kloeden, Peter E. II. Palmer, Kenneth J. (Kenneth James), 1945–. III. Deakin University. IV. Series: Contemporary mathematics (American Mathematical Society); v. 172.

QA614.8.C54 1994

515'.352—dc20

94-21580

CIP

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## Preface

Much of what we know about specific dynamical systems is obtained from numerical experiments. Not only can we visualize the dynamics, we can also evaluate key characteristics such as bifurcation parameters, Lyapunov exponents, Hausdorff dimension and entropy. The discretization process usually has no significant effect on the results for relatively simple, well-behaved dynamics. On the other hand, acute sensitivity to changes in initial conditions is a hallmark of chaotic behaviour, so how confident can we be that the numerical dynamics reflects that of the original system? Do we always obtain a nearby attractor, homoclinic orbit or inertial manifold? Do numerically calculated trajectories always shadow a true one? In brief, what role does numerical analysis have in the study of dynamical systems? Conversely, can recent advances in dynamical systems provide new insights into our understanding of numerical algorithms?

These and related issues at the interface of dynamical systems and numerical analysis were the focus of the workshop *CHAOTIC NUMERICS: an international workshop on the approximation and computation of complicated dynamical behaviour*, which took place at Deakin University in Geelong, Australia, in July 1993. Over sixty delegates, half of whom were from outside Australia, participated. There were forty long and short lectures, including some computing and video presentations. The keynote speakers (W.-J. Beyn, S.N. Chow, J. Hale, J. Lorenz) each gave two one-hour lectures, while the invited speakers (R. Corless, C.M. Elliott, H. Koçak, J.M. Sanz-Serna, T. Sauer, H. Yee) gave a one-hour lecture. The contributions in this book are based on these lectures together with several selected from those of other speakers to provide a broad overview of the different matters covered at the workshop.

While organized independently, this workshop can be seen as a natural sequel to the “*Dynamics of Numerics and Numerics of Dynamics*” conference held in Bristol in 1990. Readers may also be interested to consult its proceedings, and are strongly recommended to read Andrew Stuart’s definitive exposition and survey article “*The Numerical Analysis of Dynamical Systems*” in the 1994 issue of *Acta Numerica*.

The organizers are grateful to the Faculty of Science and Technology of Deakin University and the Australian Mathematical Society for sponsorship and financial support of the “*CHAOTIC NUMERICS*” workshop. They thank the participants for their stimulating contributions, verbal and written, and the American Mathematical Society for making these contributions more widely available by publishing this book.

*Peter Kloeden, Geelong*  
*Ken Palmer, Coral Gables*  
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## **Chaotic Numerics**

**Peter E. Kloeden and Kenneth J. Palmer, Editors**

Much of what is known about specific dynamical systems is obtained from numerical experiments. Although the discretization process usually has no significant effect on the results for simple, well-behaved dynamics, acute sensitivity to changes in initial conditions is a hallmark of chaotic behavior. How confident can one be that the numerical dynamics reflects that of the original system? Do numerically calculated trajectories always shadow a true one? What role does numerical analysis play in the study of dynamical systems? And conversely, can advances in dynamical systems provide new insights into numerical algorithms? These and related issues were the focus of the workshop on Chaotic Numerics, held at Deakin University in Geelong, Australia, in July 1993. The contributions to this book are based on lectures presented during the workshop and provide a broad overview of this area of research.

**ISBN 0-8218-5184-5**



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