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173

Geometry of the Spectrum

1993 Joint Summer Research Conference
on Spectral Geometry
July 17–23, 1993
University of Washington, Seattle

Robert Brooks
Carolyn Gordon
Peter Perry
Editors

American Mathematical Society

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This volume contains the proceedings of the 1993 AMS-IMS-SIAM Joint Summer Research Conference on Spectral Geometry which was held at the University of Washington, Seattle, on July 17–23, 1993. This conference was supported in part by the National Science Foundation and the International Science Foundation.

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Preface

Spectral geometry is, to borrow a metaphor from the contribution of Jeff Angel *et. al.*, one of the golden threads running through contemporary mathematics. Drawing from and stimulating developments in areas of mathematics as diverse as Lie algebras, graph theory, group representation theory, and Riemannian geometry, it concerns the relationship between the spectrum of the Laplace operator or its graph-theoretic analogue, the adjacency matrix, to underlying geometric and topological data. This volume collects contributions from an AMS Joint Summer Research Conference on spectral geometry held from July 17 to 23 of 1993 at the University of Washington.

The study of spectral geometry was stimulated by Marc Kac's celebrated lecture and article, "Can one hear the shape of a drum?" (*Am. Math. Monthly* **73** (1966), 1–23). If Ω is a bounded, simply connected plane domain, the eigenvalues of the Dirichlet Laplacian on Ω determine the natural frequencies of vibration of a drum with a uniform homogeneous drumhead with shape Ω under constant tension. To what extent can one invert the process and deduce the geometry of the drum from its Dirichlet eigenvalues?

Transposed to the context of Riemannian geometry, Kac's question becomes: can one reconstruct the diffeomorphism type and metric of a Riemannian manifold (M, g) from the eigenvalues of the Laplace-Beltrami operator on M? Respondents to this question may be loosely grouped into optimists, pessimists, graph theorists, and others, or, more precisely, into (1) those who use spectral invariants to prove compactness and finiteness theorems for isospectral manifolds, (2) those who use group theory and Lie algebra methods to construct isospectral, non-isometric Riemannian manifolds, (3) those who study associated problems for the Laplacian on graphs, exploiting techniques of graph theory and combinatorics, and (4) those who study the geometry of the spectrum in situations of singular interest or importance to other areas of mathematics.

A key development in the construction of isospectral manifolds was the beautiful trace formula of Sunada (*Ann. Math.* **121** (1985), 169–186). Sunada's theorem allows one to construct pairs of isospectral manifolds which have a common, finite Riemannian covering given a simple group-theoretic condition on their respective covering groups. Hubert Pesce proves a stronger version of Sunada's original theorem and addresses the converse question of whether all pairs of

isospectral manifolds with a common finite covering arise from a Sunada construction. Ruth Gornet constructs pairs of isospectral manifolds with a common infinite covering by a representation-theoretic method related to Sunada's technique and compares their geodesic length spectra. Not all isospectral closed manifolds have a common covering, however; Carolyn Gordon constructs examples of isospectral manifolds which differ in their local geometry.

Another important line of development in spectral geometry is the use of spectral invariants to show that spectral conditions determine a manifold up to finitely many topological or diffeomorphism types, and its metric up to a compact set. The prototypical result of this genre is the work of Osgood, Phillips, and Sarnak (*J. Funct. Anal.* **80** (1988), 212–234) which proved that the spectrum of the Laplacian on a compact surface constrains its metric to lie in a compact set in the C^∞ topology on positive definite metrics. The contributions of Brooks and Brooks-Glezen provide, respectively, a survey of some recent work in this field and an important bootstrap theorem for proving C^∞ compactness results from spectral data. Yaroslav Kuryiev gives some recent precise results on determining the metric and geometry of a manifold with boundary from its so-called boundary spectral data. Xianzhe Dai and Guofang Wei show that on certain "thin" subsets of the space of metrics on a Riemannian manifold of volume one, a finite number of eigenvalues suffice to determine the metric. On the other hand, Józef Dodziuk shows that there exist manifolds in dimension three or higher with metrics of volume one and arbitrarily large first eigenvalue.

Spectral geometry has been stimulated by the study of the Laplacian on graphs. Jeff Angel *et. al.* study the spectral theory of the graph Laplacian on finite analogues of the upper half-plane, with special attention to spherical functions and their ergodic properties. Koji Fujiwara proves an analogue, in the context of spectral theory on graphs, of the celebrated convergence theorem of Fukaya (*Inventiones Math.* **87** (1987), 517–547). Gregory Quenell relates the return generating function for a Cayley graph of a free product of groups to the first-return function of the factor groups, implying a relationship between the Green's function of the Laplacian on the product to the Green's functions of the factors.

Manifolds of negative curvature provide an especially attractive setting for spectral geometry. The quotient of hyperbolic two-dimensional space by the celebrated modular group $PSL(2, Z)$ provides an example of a noncompact, finite-volume surface with an infinite sequence of eigenvalues embedded in the continuous spectrum; a much-studied but as yet not completely understood question is whether such eigenvalues are "generically" absent if the metric on such a surface is varied over constant curvature metrics. Lizhen Ji and Steven Zelditch contribute to the study of this problem by showing its close relationship with multiplicity of the discrete spectrum. Yiping Mao studies conjugacy of geodesic flow on negatively curved manifolds; this work is closely related tospectral ques-

tions since geodesic flow is the "classical mechanics" for which the spectral theory of the Laplacian is the corresponding "quantum mechanics".

This relationship is further studied in the contributions of Edoh Amiran and Eugene Gutkin, who work in the context of billiards, i.e, geodesic flow on bounded planar domains. Gutkin proves an extremal property of the generating function of a twist map closely related to extremal properties of the spectrum on a bounded domain; Amiran derives a new set of spectral invariants for billiard flow by studying evolutes of the underlying bounded domain.

The contributions of Yiannis Petridis, Leonid Malozemov, and Toshikazu Sunada illustrate the rich variety of connections between spectral geometry and other branches of pure and applied mathematics. Malozemov studies the asymptotic distribution of eigenvalues for the Laplacian on certain fractal domains. Petridis shows that normalized differences of eigenvalues on generic flat tori are dense in $\mathbf{R}$, a statistical property of current interest to students of "quantum chaos". Sunada investigates a family of difference operators on graphs which provide a discrete analogue for Schrödinger operators in magnetic fields.

The editors would like to express their appreciation to all of the participants in this conference for making it such a lively and stimulating experience. We would also like to thank Carole Kohanski and Chris Harkness of the American Mathematical Society, whose diligence and competence made our administrative task, to use a favorite mathematical phrase, completely trivial. Donna Harmon, our editorial assistant at the American Mathematical Society, displayed unusual patience dealing with three editors who were in three different countries for most of the year. Finally, we appreciate the forbearance of our tour guide at a local winery, who couldn't understand at first why we all laughed when he proudly announced that this winery was one of the few in the region with its own "manifold"—in this case, a system of pipes that facilitates the many chemical reactions that turn crushed grapes into wine!

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Spectral geometry runs through much of contemporary mathematics, drawing on and stimulating developments in such diverse areas as Lie algebras, graph theory, group representation theory, and Riemannian geometry. The aim is to relate the spectrum of the Laplace operator or its graph-theoretic analogue, the adjacency matrix, to underlying geometric and topological data. This volume brings together papers presented at the AMS-IMS-SIAM Joint Summer Research Conference on Spectral Geometry, held in July 1993 at the University of Washington in Seattle. With contributions from some of the top experts in the field, this book presents an excellent overview of current developments in spectral geometry.

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