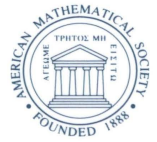
CONTEMPORARY MATHEMATICS

176

Foundational Aspects of "Non"standard Mathematics

David Ballard



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ABSTRACT. Early in the development of Nonstandard Analysis Luxemburg noted natural topologies (the "S-topologies") to exist on the internal part of a Robinson enlargement. In this work these are generalized and used to give new, topological foundations for Nonstandard Mathematics. The resulting topological methods are then applied to construct models (implying conservativity over ZFC) of the nonstandard set theories proposed by Nelson, Hrbáček and Kawai. A simple yet nontrivial extension of a nonstandard set theory of Fletcher's is then described and proposed as a prototype of the "ultimate" vehicle for Nonstandard Mathematics. Although the mathematical environment it presents is radically relativistic, it is never the less shown to be "safe" (conservative over ZFC) for practitioners.

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Contents

Foundational Aspects of "Non"standard Mathematics
David Ballard

This work proposes a major new extension of "non"standard mathematics. Addressed to a general mathematical audience, the book is intended to be philisophically provocative. The model theory on which "non"standard mathematics has been based is first reformulated within point set topology, which facilitates proofs and adds perspective. These topological techniques are then used to give new, uniform conservativity proofs for the various versions of "non"standard mathematics proposed by Nelson, Hrbáček, and Kawai. The proofs allow for sharp comparison. Addressing broader issues, Ballard then argues that what is novel in these forms of "non"standard mathematics is the introduction, however tentative, of relativity in one's mathematical environment. This hints at the possibility of a mathematical environment which is radically relativistic. The work's major and final feature is to present and prove conservative a version of "non"standard mathematics which, for the first time, illustrates this full radical relativism. The book is entirely self-contained, with all necessary background in point set topology, model theory, "non"standard analysis, and set theory provided in full.

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