Algebraic $K$-Theory

Conference on Algebraic $K$-Theory
September 4–8, 1995
The Adam Mickiewicz University
Poznań, Poland

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This volume contains the refereed proceedings of the research conference on algebraic $K$-theory that took place in Poznań, Poland, on September 4–8, 1995. The conference concluded the activity of the algebraic $K$-theory seminar held at the Adam Mickiewicz University in 1994–1995.

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Preface

This book is a result of the research conference on algebraic $K$-theory which took place in Poznań, POLAND in September 1995. The conference concluded the activity of algebraic $K$-theory seminar held at the Adam Mickiewicz University in the academic year 1994/95. Talks at the conference covered wide range of current research activities in algebraic $K$-theory. In particular, the following topics were covered at the conference:

1. $K$-theory of fields and rings of integers,
2. $K$-theory of elliptic and modular curves,
3. Theory of motives, motivic cohomology, Beilinson conjectures,
4. Algebraic $K$-theory of topological spaces, topological Hochschild homology and cyclic homology.

We would like to thank all participants for their contribution to make the conference successful. We are especially thankful to the authors for submitting their papers to these proceedings.

We are also grateful to the Board of Regents of the Adam Mickiewicz University, the Board of Regents of Szczecin University, and Dean of Mathematics Department of the Adam Mickiewicz University for their support. Finally, we wish to acknowledge the financial support of the following institutions: Polish Committee for Scientific Research (KBN), Mayor of the City of Poznań, Telekomunikacja Polska S.A., Bank PEKAO S.A. and MTP S.A..

Grzegorz Banaszak
Wojciech Gajda
Piotr Krason

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Monday, September, 4th

9:00-10:00 DANIEL QUILLEN
Module theory and K-theory for nonunital rings

10:30-11:30 MARIUSZ WODZICKI
Algebraic K-theory of operator ideals

11:45-12:45 PETER SCHNEIDER
The cyclic homology of p-adic reductive groups

14:30-15:00 ROBERT PERLIS
Tame equivalence and the 2-rank of tame kernels

15:15-15:45 JERZY BROWKIN
On the p-rank of the tame kernel of number fields

16:00-16:30 BERNHARD KÖCK
Adams operations for projective modules over group rings

16:45-17:15 PHILLIPE ELBAZ-VINCENT
Homology of GL and SL in low degree and $K_3$

Tuesday, September, 5th

9:00-10:00 DANIEL BURGHELEA
Free loop spaces, K-theory and epicyclic structures

10:30-11:30 ZBIGNIEW FIEDOROWICZ
Iterated loop spaces and iterated monoidal categories

11:45-12:45 RAINER VOGT
"Brave new algebra" and MacLane homology

14:30-15:00 ROLAND SCHWÄNZL
Co-multiplication and suspension

15:15-15:45 STANISLAW BETLEY
Stable K-theory with coefficients

16:00-16:30 CHARLES GIFFEN
Bott periodicity via algebraic K-theory

16:45-17:15 JAN SPALIŃSKI
Homotopy theory of dihedral sets
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**Wednesday, September, 6th**

9:00-10:00 DOMINIQUE ARLETTAZ
*Some remarks on the homotopy type of the algebraic K-theory space BE(R)^+*

10:30-11:30 J.F. JARDINE
*Descent problems for the K-theory of fields of Galois cohomological dimension one*

**Thursday, September, 7th**

9:00-10:00 SPENCER BLOCH
*An Atiyah-Hirzebruch spectral sequence in algebraic K-theory*

14:30-15:00 JÖRG WILDESHAUS
*On an elliptic analogue of Zagier’s conjecture*

11:45-12:45 ANTHONY SCHOLL
*K-theory, motives, and special values of incomplete L-functions*

14:30-15:00 STEFAN JACKOWSKI
*The Grothendieck ring of vector bundles over classifying spaces of compact Lie groups*

15:15-15:45 ANDREAS LANGER
*Local points of motives in semistable reduction*

16:00-16:30 ANDRZEJ DĄBROWSKI
*The symmetric power of a newform : Special L-values and Iwasawa theory*

16:45-17:15 HERBERT GANGL
*Families of functional equations for polylogarithms*

**Friday, September, 8th**

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*Operations on algebraic and arithmetic K-theory*

10:30-11:30 BRUNO KAHN
*Unramified cohomology of quadrics*

11:45-12:45 MANFRED KOLSTER
*Higher regulators for number fields and special values of L-functions*

14:30-15:00 MAREK SZYJEWSKI
*Witt ring of a Grassmann variety*

15:15-15:45 DAMIAN ROESSLER
*Adams operations for the K-theory of hermitian bundles*
16:00-16:30 MAREK GOLASIŃSKI

\( K_0 \) of 0-forms on a finite simplicial \( G \)-complex

16:45-17:15 ALEXANDER NENASHEV

On Quillen’s \( K_1 \) of a non-splitting exact category
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Algebraic $K$-Theory
Grzegorz Banaszak, Wojciech Gajda, and Piotr Krasoń, Editors

This book contains proceedings of the research conference on algebraic $K$-theory that took place in Poznań, Poland, in September 1995. The conference concluded the activity of the algebraic $K$-theory seminar held at the Adam Mickiewicz University in the academic year 1994–1995. Talks at the conference covered a wide range of current research activities in algebraic $K$-theory. In particular, the following topics were covered:

- $K$-theory of fields and rings of integers.
- $K$-theory of elliptic and modular curves.
- Theory of motives, motivic cohomology, Beilinson conjectures.
- Algebraic $K$-theory of topological spaces, topological Hochschild homology and cyclic homology.

With contributions by some leading experts in the field, this book provides a look at the state of current research in algebraic $K$-theory.