

CONTEMPORARY MATHEMATICS

216

Wavelets, Multiwavelets, and Their Applications

AMS Special Session on Wavelets,
Multiwavelets, and Their Applications
January 1997
San Diego, California

Akram Aldroubi
EnBing Lin
Editors



Selected Titles in This Series

- 216 **Akram Aldroubi and EnBing Lin, Editors**, Wavelets, multiwavelets, and their applications, 1998
- 215 **M. G. Nerurkar, D. P. Dokken, and D. B. Ellis, Editors**, Topological dynamics and applications, 1998
- 214 **Lewis A. Coburn and Marc A. Rieffel, Editors**, Perspectives on quantization, 1998
- 213 **Farhad Jafari, Barbara D. MacCluer, Carl C. Cowen, and A. Duane Porter, Editors**, Studies on composition operators, 1998
- 212 **E. Ramírez de Arellano, N. Salinas, M. V. Shapiro, and N. L. Vasilevski, Editors**, Operator theory for complex and hypercomplex analysis, 1998
- 211 **Józef Dodziuk and Linda Keen, Editors**, Lipa's legacy: Proceedings from the Bers Colloquium, 1997
- 210 **V. Kumar Murty and Michel Waldschmidt, Editors**, Number theory, 1998
- 209 **Steven Cox and Irena Lasiecka, Editors**, Optimization methods in partial differential equations, 1997
- 208 **Michel L. Lapidus, Lawrence H. Harper, and Adolfo J. Rumbos, Editors**, Harmonic analysis and nonlinear differential equations: A volume in honor of Victor L. Shapiro, 1997
- 207 **Yujiro Kawamata and Vyacheslav V. Shokurov, Editors**, Birational algebraic geometry: A conference on algebraic geometry in memory of Wei-Liang Chow (1911–1995), 1997
- 206 **Adam Korányi, Editor**, Harmonic functions on trees and buildings, 1997
- 205 **Paulo D. Cordaro and Howard Jacobowitz, Editors**, Multidimensional complex analysis and partial differential equations: A collection of papers in honor of François Trèves, 1997
- 204 **Yair Censor and Simeon Reich, Editors**, Recent developments in optimization theory and nonlinear analysis, 1997
- 203 **Hanna Nencka and Jean-Pierre Bourguignon, Editors**, Geometry and nature: In memory of W. K. Clifford, 1997
- 202 **Jean-Louis Loday, James D. Stasheff, and Alexander A. Voronov, Editors**, Operads: Proceedings of Renaissance Conferences, 1997
- 201 **J. R. Quine and Peter Sarnak, Editors**, Extremal Riemann surfaces, 1997
- 200 **F. Dias, J.-M. Ghidaglia, and J.-C. Saut, Editors**, Mathematical problems in the theory of water waves, 1996
- 199 **G. Banaszak, W. Gajda, and P. Krasoń, Editors**, Algebraic K -theory, 1996
- 198 **Donald G. Saari and Zhihong Xia, Editors**, Hamiltonian dynamics and celestial mechanics, 1996
- 197 **J. E. Bonin, J. G. Oxley, and B. Servatius, Editors**, Matroid theory, 1996
- 196 **David Bao, Shiing-shen Chern, and Zhongmin Shen, Editors**, Finsler geometry, 1996
- 195 **Warren Dicks and Enric Ventura**, The group fixed by a family of injective endomorphisms of a free group, 1996
- 194 **Seok-Jin Kang, Myung-Hwan Kim, and Insok Lee, Editors**, Lie algebras and their representations, 1996
- 193 **Chongying Dong and Geoffrey Mason, Editors**, Moonshine, the Monster, and related topics, 1996
- 192 **Tomek Bartoszyński and Marion Scheepers, Editors**, Set theory, 1995
- 191 **Tuong Ton-That, Kenneth I. Gross, Donald St. P. Richards, and Paul J. Sally, Jr., Editors**, Representation theory and harmonic analysis, 1995
- 190 **Mourad E. H. Ismail, M. Zuhair Nashed, Ahmed I. Zayed, and Ahmed F. Ghaleb, Editors**, Mathematical analysis, wavelets, and signal processing, 1995
- 189 **S. A. M. Marcantognini, G. A. Mendoza, M. D. Morán, A. Octavio, and W. O. Urbina, Editors**, Harmonic analysis and operator theory, 1995

(Continued in the back of this publication)

Wavelets, Multiwavelets, and Their Applications

CONTEMPORARY MATHEMATICS

216

Wavelets, Multiwavelets, and Their Applications

AMS Special Session on Wavelets,
Multiwavelets, and Their Applications
January 1997
San Diego, California

Akram Aldroubi
EnBing Lin
Editors



American Mathematical Society
Providence, Rhode Island

Editorial Board

Dennis DeTurck, managing editor

Andy Magid
Clark Robinson

Michael Vogelius
Peter M. Winkler

1991 *Mathematics Subject Classification*. Primary 42A15, 41A25, 42A38, 42A10, 42C15, 41A58, 42A99, 42C99;
Secondary 43A99, 39B62, 65D99, 46E40, 94A11, 94A12, 46C50, 45B05, 46N30.

Library of Congress Cataloging-in-Publication Data

AMS Special Session on Wavelets, Multiwavelets, and Their Applications (1997 : San Diego, Calif.)

Wavelets, multiwavelets, and their applications : AMS Special Session on Wavelets, Multiwavelets, and Their Applications, January 1997, San Diego, California / Akram Aldroubi, EnBing Lin, editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 216)

Includes bibliographical references.

ISBN 0-8218-0793-5 (alk. paper)

1. Wavelets (Mathematics)—Congresses. I. Aldroubi, Akram. II. Lin, EnBing, 1953– III. Title. IV. Series: Contemporary mathematics (American Mathematical Society) ; v. 216.

QA403.3.A45 1997

515'.2433—dc21

97-38981

CIP

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Assistant to the Publisher, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to reprint-permission@ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 1998 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights
except those granted to the United States Government.
Printed in the United States of America.

∞ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.

Visit the AMS home page at URL: <http://www.ams.org/>

10 9 8 7 6 5 4 3 2 1 03 02 01 00 99 98

Contents

Part I. Wavelet Theory and Applications

Extensions of no-go theorems to many signal systems RADU BALAN	3
Wavelet sets in \mathbb{R}^n . II XINGDE DAI, DAVID R. LARSON, AND DARRIN M. SPEEGLE	15
An analogue of Cohen's condition for nonuniform multiresolution analyses JEAN-PIERRE GABARDO AND M. ZUHAIR NASHED	41
Positive estimation with wavelets GILBERT G. WALTER AND XIAOPING SHEN	63
A class of quasi-orthogonal wavelet bases RICHARD A. ZALIK	81

Part II. Multiwavelet Theory and Applications

Characterization and parameterization of multiwavelet bases AKRAM ALDROUBI AND MANOS PAPADAKIS	97
Nonhomogeneous refinement equations THOMAS B. DINSENBACHER AND DOUGLAS P. HARDIN	117
Multi-scaling function interpolation and approximation ENBING LIN AND ZHENGCHU XIAO	129
A note on construction of biorthogonal multi-scaling functions VASILY STRELA	149
Orthonormal matrix valued wavelets and matrix Karhunen-Loève expansion XIANG-GEN XIA	159

Preface

This volume contains refereed research articles on wavelets and multiwavelets. It draws upon research presented in the special session on Wavelets, Multiwavelets, and Their Applications held at the annual American Mathematical Society meeting which took place in San Diego, California in January 1997.

This book is divided into two parts: (1) Wavelet theory and applications; (2) Multiwavelet theory and applications. The first part consists of a collection of new results on the classical theory of wavelets in which the wavelet spaces are generated by dilations and translations of a single function. The second part is devoted to the theory of multiwavelets in which the wavelet spaces are generated by the dilations and translations of several intertwining functions. This added complexity gives us more flexibility for the construction of wavelet bases with some desired shape and/or properties.

We would like to thank the referees for their professional work. We are also grateful to Ms. Christine M. Thivierge and Dr. Sergei I. Gelfand for their help at every phase of this venture. Finally, we thank Professor Dennis DeTurck and the American Mathematical Society Editorial Board for their efficient supervision to produce this volume.

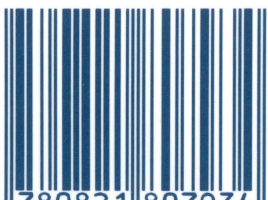
Selected Titles in This Series

(Continued from the front of this publication)

- 188 **Alejandro Adem, R. James Milgram, and Douglas C. Ravenel, Editors**, Homotopy theory and its applications, 1995
- 187 **G. W. Brumfiel and H. M. Hilden, $SL(2)$ representations of finitely presented groups**, 1995
- 186 **Shreeram S. Abhyankar, Walter Feit, Michael D. Fried, Yasutaka Ihara, and Helmut Voelklein, Editors**, Recent developments in the inverse Galois problem, 1995
- 185 **Raúl E. Curto, Ronald G. Douglas, Joel D. Pincus, and Norberto Salinas, Editors**, Multivariable operator theory, 1995
- 184 **L. A. Bokut', A. I. Kostrikin, and S. S. Kutateladze, Editors**, Second International Conference on Algebra, 1995
- 183 **William C. Connett, Marc-Olivier Gebuhrer, and Alan L. Schwartz, Editors**, Applications of hypergroups and related measure algebras, 1995
- 182 **Selman Akbulut, Editor**, Real algebraic geometry and topology, 1995
- 181 **Mila Cenkli and Haynes Miller, Editors**, The Čech Centennial, 1995
- 180 **David E. Keyes and Jinchao Xu, Editors**, Domain decomposition methods in scientific and engineering computing, 1994
- 179 **Yoshiaki Maeda, Hideki Omoro, and Alan Weinstein, Editors**, Symplectic geometry and quantization, 1994
- 178 **Hélène Barcelo and Gil Kalai, Editors**, Jerusalem Combinatorics '93, 1994
- 177 **Simon Gindikin, Roe Goodman, Frederick P. Greenleaf, and Paul J. Sally, Jr., Editors**, Representation theory and analysis on homogeneous spaces, 1994
- 176 **David Ballard**, Foundational aspects of "non"standard mathematics, 1994
- 175 **Paul J. Sally, Jr., Moshe Flato, James Lepowsky, Nicolai Reshetikhin, and Gregg J. Zuckerman, Editors**, Mathematical aspects of conformal and topological field theories and quantum groups, 1994
- 174 **Nancy Childress and John W. Jones, Editors**, Arithmetic geometry, 1994
- 173 **Robert Brooks, Carolyn Gordon, and Peter Perry, Editors**, Geometry of the spectrum, 1994
- 172 **Peter E. Kloeden and Kenneth J. Palmer, Editors**, Chaotic numerics, 1994
- 171 **Rüdiger Göbel, Paul Hill, and Wolfgang Liebert, Editors**, Abelian group theory and related topics, 1994
- 170 **John K. Beem and Krishan L. Duggal, Editors**, Differential geometry and mathematical physics, 1994
- 169 **William Abikoff, Joan S. Birman, and Kathryn Kuiken, Editors**, The mathematical legacy of Wilhelm Magnus, 1994
- 168 **Gary L. Mullen and Peter Jau-Shyong Shiue, Editors**, Finite fields: Theory, applications, and algorithms, 1994
- 167 **Robert S. Doran, Editor**, C^* -algebras: 1943–1993, 1994
- 166 **George E. Andrews, David M. Bressoud, and L. Alayne Parson, Editors**, The Rademacher legacy to mathematics, 1994
- 165 **Barry Mazur and Glenn Stevens, Editors**, p -adic monodromy and the Birch and Swinnerton-Dyer conjecture, 1994
- 164 **Cameron Gordon, Yoav Moriah, and Bronislaw Wajnryb, Editors**, Geometric topology, 1994
- 163 **Zhong-Ci Shi and Chung-Chun Yang, Editors**, Computational mathematics in China, 1994
- 162 **Ciro Ciliberto, E. Laura Livorni, and Andrew J. Sommese, Editors**, Classification of algebraic varieties, 1994
- 161 **Paul A. Schweitzer, S. J., Steven Hurder, Nathan Moreira dos Santos, and José Luis Arraut, Editors**, Differential topology, foliations, and group actions, 1994
- 160 **Niky Kamran and Peter J. Olver, Editors**, Lie algebras, cohomology, and new applications to quantum mechanics, 1994

(See the AMS catalog for earlier titles)

ISBN 0-8218-0793-5



9 780821 807934

AMS *on the Web*
www.ams.org