Monge Ampère Equation: Applications to Geometry and Optimization

NSF-CBMS Conference on The Monge Ampère Equation: Applications to Geometry and Optimization
July 9–13, 1997
Florida Atlantic University

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Preface

In July 1997, a CBMS Conference on “Monge Ampère Equation: Applications to Geometry and Optimization” took place at Deerfield Beach, Florida, with the support of NSF, and Florida Atlantic University, Broward Campus. Luis Caffarelli gave a 10 lecture course and distinguished specialists from all over the world gave invited lectures.

In recent years the Monge Ampère Equation, a classical equation from geometry and physics, has received a lot of attention for its role in several new areas of applied mathematics:

- As a new method of discretization for evolution equations of classical mechanics, like Euler equation, flow in porous media, Hele Shaw flow.

- As a simple model for optimal transportation and a div-curl decomposition with affine invariance.

- As a model for front formation in meteorology and optimal antenna design.

Many of the lecturers addressed these applications as well as other important theoretical developments for equations of Monge Ampère type. Professor Caffarelli’s lecture notes will appear in the CBMS Lecture Note Series but we felt it was important to collect the invited lectures and contributions in a separate volume, not only for the high quality of the research presented, but also because they represent a new important development in nonlinear analysis and its applications.

We would like to thank the participants and contributors to this volume for the effort they put in making the conference (and we hope this volume) an interesting, successful event.

We are also grateful to Mary McBride, Vice President, F.A.U., Broward Campus, John Wiesenfeld, Dean, College of Science, F.A.U., and H. Niederhausen, Chair, Department of Mathematics, F.A.U., for their support to the conference; to Meike Niederhausen and Vanessa Fonseca Milman Gomes, who acted as secretaries of the conference providing invaluable help in solving all the open problems. Last but not least we thank Vanda Milman for her help in the organization and execution of all the social functions.

Luis A. Caffarelli, Austin, Texas, July 1998
Mario Milman, Delray Beach, Florida, July 1998
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