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Higher Category Theory
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Workshop on Higher Category Theory and Physics
March 28–30, 1997
Northwestern University
Evanston, IL

Ezra Getzler
Mikhail Kapranov
Editors
This volume is the record of a Workshop on Higher Category Theory and Physics, which took place at Northwestern University, Evanston, IL, on March 28–30, 1997.

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Preface

The concept of categories, introduced by Eilenberg and Mac Lane in the 1940's, has replaced set theory as the foundation in a large part of mathematics. One reason is that more information is retained if one considers objects in categories rather than their underlying sets. To put it more concisely: look at a set through a magnifying glass and you see a category. The passage from sets to categories involves a change in point of view: in most cases, it is neither meaningful nor necessary to speak about two objects of a category being equal\(^1\); much better to say that they are isomorphic, and specify the isomorphism.

Soon after the introduction of categories, it was realized that the Hom-sets in a category are frequently in their turn categories, and so on. Iterating \(n\) times, one obtains the theory of \(n\)-categories. A 0-category is just a set, a 1-category is the same as an ordinary category and an example of a 2-category is provided by the “category” of all 1-categories. However, it turned out to be very difficult to give an “invariant” definition of an \(n\)-category (i.e. a definition which at no stage involves the concept of equality of two objects of an ordinary category). This problem was explicitly stated by Grothendieck in his unpublished memoir “Pursuing stacks” (1986). A naive attempt to write down a definition stage by stage quickly leads to seemingly unsurmountable combinatorial difficulties.

In recent years, several definition of (weak) \(n\)-categories have been proposed [1,2,3,4]. One reason for increased interest in the subject was the discovery of applications in low-dimensional topology and mathematical physics, where monoidal categories (2-categories with one object) have been used for over a decade, and where the recent work has led to the consideration of higher categorical structures.

This volume is the record of the workshop on higher category theory and mathematical physics held at Northwestern University in March 1997 with the aim of making some of these exciting new developments in category theory better known outside the community of experts, as they deserve to be. We especially tried to solicit presentations in the style “Higher categories for the working mathematician.” We hope that the present collection of papers will be useful as an introduction to a subject which is intrinsically difficult but holds great promise for the future development of mathematics.

Ezra Getzler, Misha Kapranov
October 1998, Evanston

\(^1\)Compare to these maxims of Wittgenstein: “To say of two things that they are identical is nonsense and to say that one thing is identical with itself is to say nothing at all (…) The identity-sign, therefore, is not an essential constituent of conceptual notation” ([5], §5.53).
Bibliography


