Continued Fractions: From Analytic Number Theory to Constructive Approximation

A Volume in Honor of L. J. Lange

May 20-23, 1998
University of Missouri—Columbia

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Continued Fractions:
From Analytic Number Theory
to Constructive Approximation

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May 20–23, 1998
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Fritz Gesztesy
Editors

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This volume contains the contributions to the international conference "Continued Fractions: From Analytic Number Theory to Constructive Approximation", held at the University of Missouri-Columbia on May 20-23, 1998.

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Preface and Dedication

This volume contains the refereed contributions of the international conference “Continued Fractions: From Analytic Number Theory to Constructive Approximation”, held at the University of Missouri–Columbia on May 20–23, 1998. The meeting also celebrated Jerry Lange’s seventieth birthday and marked his retirement from the University of Missouri. It is a great pleasure to dedicate this volume to Jerry in recognition of his distinguished service and long-lasting impact on the Mathematics Department at MU.

In spite of their long tradition, continued fractions (whose general definition appears to go back to the book “Liber Abaci” of Leonardo of Pisa, also called Fibonacci, written around 1202) remain an active area of research in a large number of fields ranging from pure mathematics to mathematical physics and approximation theory. The principal purpose of this conference was to focus on continued fractions as a common interdisciplinary theme bridging gaps between these fields. As a consequence, the lectures at this conference and the corresponding contributions to this volume reflect the wide range of applicability of continued fractions in mathematics and the applied sciences.

More specifically, recurrence relations for orthogonal polynomials and their relations to continued fractions, appealing to a well-known theorem of Markoff, are studied in Askey’s contribution. Orthogonal Laurent polynomials and questions of (in)determinacy of the strong Stieltjes moment problem are considered by Njastad. Strong Stieltjes distributions, orthogonal Laurent polynomials and related continued fractions appear in Bracciali’s article. Generating functions of orthogonal Laurent polynomials, associated kernel polynomials, and moment-preserving approximations are treated by Bojanov and Sri Ranga. Convergence properties of Laurent polynomials that interpolate functions on the boundary of a circular annulus are investigated by Li. Compact perturbations of reflection coefficients and the resulting weak asymptotics of orthogonal polynomials on the support of the measure of orthogonality and asymptotics of the corresponding functions of the second kind are studied by Peherstorfer and Steinbauer. Convergence of Stieltjes continued fractions and their asymptotic speed of convergence as a result of analyzing the asymptotic behavior of the corresponding expansion coefficients, applicable to Whittaker functions, are considered by Jones and Shen. Various convergence theorems including a new and constructive proof of an extension of Van Vleck’s convergence theorem for continued fractions, including a sharp truncation error formula, are derived in Lange’s contribution. A very extensive treatment of convergence criteria, including substantial progress in proving a conjecture by Lorentzen and Ruscheweyh, as well as a simplified approach to Cordova’s extension of the classical parabola theorem, are provided by Lorentzen.
PREFACE AND DEDICATION

A thorough survey of all 58 problems submitted by Ramanujan to the *Journal of the Indian Mathematical Society* between 1911 and 1919, detailing their subsequent influence on various directions of research since 1927, is provided by Berndt, Choi, and Kang. Recent progress made in explicit evaluations of the Rogers-Ramanujan continued fractions using modular equations and class invariants, are discussed by Chan and Tan. Continued fractions of Stieltjes and Rogers associated with elliptic functions and solutions of the corresponding difference equations are studied by Ismail and Masson.

A sequence of approximation constants converging towards $1/3$ by lower values associated with the Diophantine equation $x^2 + y^2 + z^2 = 3xyz + 2x$ is developed by Perrine. A principle relating the reduction mod $p$ of a continued fraction expansion to the original expansion in characteristic zero and illustrations involving classical Abelian integrals and hyperelliptic curves are discussed by van der Poorten. Multiple orthogonal polynomials, a notion closely related to Hermite-Padé rational approximation of a system of Markoff functions, and constructive proofs of the irrationality of $\zeta(3)$ and transcendence of certain real numbers (applicable to $e$) are presented by Van Assche. Convergence questions and the location of zeros of Hermite-Padé approximants to $e^z$ are considered by Wielonsky.

Orthogonal reproducing rational kernels are used to construct rational wavelets on the unit circle with respect to arbitrary positive measures by Bultheel and González-Vera. The rate of convergence for $n$-point Gauss-type quadrature integrals on the half-line is estimated by Bultheel, Díaz-Mendoza, González-Vera, and Orive. A new approach to frequency analysis for trigonometric signals, circumventing the associated problem of vanishing Töplitz determinants, is presented by Waadeland.

The absence of pressure-dependent phase transitions in modified two-component plasmas, extending Lenard’s original approach based on continued fractions, is shown by Chelst. Spatial regularity properties of the fundamental solution (propagator) of the time-dependent Schrödinger equation on the circle in the presence of a complex-valued potential in terms of Besov spaces, depending on arithmetic properties of the time parameter, are derived by Rodnianski.

Naturally, this event required support by a number of individuals. We thank the remaining organizing committee members, Nigel Kalton, Igor Verbitsky, and especially Mark Ashbaugh for their assistance in preparing this conference. Special thanks are due to all staff members at the Mathematics Department, especially to Bridget Kelley and some of our graduate students for their great efforts on behalf of this conference. Moreover, we also thank Helge Holden and Karl Unterkofler for their help over the past two years. Holden generously shared his considerable experience in organizing events of this type; Unterkofler handled all technical aspects in producing this proceedings volume and provided invaluable assistance to several contributors and the editors.

Finally, we gratefully acknowledge financial support for this conference from a variety of sources, including the US National Science Foundation (NSF DMS 9729700), the University of Missouri Research Board (RB98-012), The College of Arts & Science, The Office of Research, and the Department of Mathematics at MU. Special thanks are due to Elias Saab, who early on enthusiastically supported the idea of such an event and, in his position as Chair of the Mathematics Department, initially guaranteed a major portion of funding for this project.
L. J. Lange
Leo Jerome Lange (Jerry) was the first of five children born to Leo and Clara Lange on August 29, 1928, in New Rockford, North Dakota. For most of his boyhood years, Jerry was raised on a farm about eleven miles from New Rockford. This farm was homesteaded by his grandparents, and it was the birthplace of his father. There he learned the value of hard work and experienced some of the adversity brought on by the aftermath of the Great Depression and the dust bowl era of the thirties.

Jerry's first eight years in school were spent in a one-room country school house. He was fortunate to have several inspiring teachers in this school and recalls reading most of the books (including the encyclopedias) in its limited library.

Jerry spent his four high school years, 1942-1946, at Sacred Heart Academy (SHA) (now Schanley High School) in Fargo, North Dakota. To continue to attend this premier high school (about 175 miles from the family home) he lived and worked in a hospital in Fargo during his last three years at the school. To this day he much appreciates the education he received from excellent dedicated teachers at SHA. He has an especially fond memory of a nun who had the uncanny ability of getting teen-age boys enthused about presenting proofs of propositions on the blackboard.

Upon graduation from high school at the rank of Salutatorian of his class, Jerry now faced the problems of how to finance his desired college education and what to do about the fact that he was about to be caught up in the post World War II military draft. The possible educational benefits of the WW II GI Bill influenced him to enlist in the US Army on September 30, 1946 for an eighteen month term. He served as a medic for more than a year in Berlin, Germany in the 3rd. Battalion, 16th Infantry Regiment, of the 1st. Division, and obtained the final rank of Corporal.

Through the benefits of the GI Bill, by working in the summer, and with some extra “travel money” from his parents, Jerry was able to attend Regis College (now Regis University) in Denver, Colorado. He started his training there in the fall of 1948 and eventually decided to major in Mathematics and minor in Physics and Philosophy. He is very appreciative of the challenges that were presented to him and the attitudes that were fostered in him by his professors at Regis. Jerry received the Most Outstanding Senior Award from Regis, and graduated with a BS degree in the spring of 1952.

Jerry had been encouraged by his mathematics teachers at Regis to pursue an advanced degree in mathematics, and in August of 1952 he went to the University of Colorado-Boulder (CU) to seek admission as a graduate student in mathematics. Burton W. Jones, who was then Chair of the Mathematics Department at CU,
accepted him immediately. Jerry began his teaching career in the winter semester of 1953 when he received a Teaching Assistantship in Mathematics.

In June of 1955, he married Geraldine Ryan who was teaching music in the Denver public school system. They lived in Denver and Jerry commuted to Boulder daily to continue his study and teaching at CU. By the end of the summer of 1955 he had completed his Masters Thesis on “Some Problems in Interpolation” and all of the requirements for the MA degree. His Masters Thesis advisor was Kurt A. Hirsch of the University of London, and the second reader for the work was S. Chowla.

In 1956 Geraldine and Jerry had started a family, so she chose to relinquish her teaching position in Denver. Jerry accepted a classified position as a mathematician at the National Bureau of Standards, Boulder Laboratories (NBS) in July of 1956. His position at NBS was located in the Low Frequency and Very Low Frequency Section of the Radio Communication and Systems Division. For the next two years he continued to take graduate courses at CU while working full-time at NBS, making up the time he spent in class by working on weekends. Jerry did most of the analyses and prepared most of the technical materials that were used by the US delegation to the International Technical Discussions on Detection of Nuclear Tests held in Geneva, Switzerland in the summer of 1958. For his work at NBS, Jerry received two merit awards from the United States Department of Commerce, one in 1959 and one in 1960.

In the fall of 1958, through his Ph.D. advisor Wolfgang J. Thron, Jerry was awarded a two-year AFOSR grant for graduate research. This allowed him to convert his position at NBS to half-time, so he could spend his mornings at CU attending classes and doing research for his doctoral thesis. Jerry was the first of Thron’s many Ph.D. students, and he was awarded the Ph.D. degree from CU in the summer of 1960. His doctoral thesis consists of three parts and is entitled “Divergence, Convergence, and Speed of Convergence of Continued Fractions 1 + K(\(a_n/1\))”. The Convergence part appears in his joint 1960 publication with Thron in Mathematische Zeitschrift, and the other two parts have played significant roles in some of Jerry’s later publications. Upon receiving the Ph.D., he was offered a large increase in civil service rank if he would continue his work at NBS. However, he chose to resign at NBS in August, 1960 so that he could accept an academic position at the University of Missouri-Columbia (MU).

Jerry joined the faculty of the Department of Mathematics at MU on September 1, 1960. He has spent his entire academic career at this institution. He served as Chair of the Department during the years 1988-91 and during the summers of 1966, 1968, and 1969. He also served in the position of Associate Chair during the years 1968-69, 1977-78, and 1978-79. His teaching record shows that he taught in the neighborhood of one hundred and ninety classes at MU, most of which were at the calculus level and beyond. He was a pioneer in the use of the computer as a teaching aid in calculus, and for three decades he was the principal creator, teacher, and promoter of complex analysis courses in the Department. Jerry gave unselfishly of his time to numerous important Departmental and Campus personnel, academic, and policy committees. He is especially proud of his role in the Departmental Planning Committee that had much to do with charting the path of the Department in the last decade.
Jerry’s principal field of research has been the analytic theory of continued fractions. He is an internationally recognized expert on the convergence of continued fractions and their applications to function representation in the complex domain. In each of his papers on continued fractions he has solved one or more, usually long-standing, problems in the field. Space limitations prohibit a discussion of Jerry’s research in detail here, but the majority of his results are fundamental, have wide-reaching significance, and are cited often by other researchers. His Uniform Twin Limaçon Theorem for continued fractions $K(a_n/1)$, which he proved in his 1966 paper in the Illinois Journal of Mathematics, has turned out to be one of the most significant results in convergence region theory. Recently, in the Campinas Proceedings, he settled a twenty-eight-year-old conjecture of Jones and Thron by proving that almost all twin convergence regions for $K(a_n/1)$ generated by either disk-disk, disk-halfplane, or disk-complement of disk mappings are embeddable in those of his Limaçon Theorem. In two papers in the early 1980’s, Jerry developed the theory of $\delta$-fractions. His $\delta$-fraction work solved a number of problems dealing with the association of continued fractions and power series for analytic functions. Others have used his $\delta$-fractions to approximate solutions of Riccati differential equations, and Jerry applied them to zero location and stability problems in one of his 1986 papers. In his 1986 work with Kalton, many outstanding problems in the area of equimodular limit-periodic continued fractions were solved. More than ten years later, some of this work was used to disprove an assertion of Ramanujan and now appears in Berndt’s Ramanujan’s Notebooks, Part V. The Worpitzky and Transcendental strips in his 1994 paper on strip convergence regions for continued fractions amount to the first known best convergence regions that are distortions of the famous parabolic regions and have inspired others to seek generalizations of them. In his 1995 paper in Constructive Approximation, Jerry completely settles the Oval Theorem for continued fractions and proves an earlier result of others false by actually showing that these ovals are embeddable in the parabolas of the Uniform Parabola Theorem. His Uniform Twin Parabola Theorem of 1994 in the Journal of Mathematical Analysis and Applications generalizes work of Thron fifty years earlier in several important ways. Uniformity and speed of convergence are obtained, and only the even elements need to be bounded. His work on Van Vleck fractions $K(1/b_n)$ in this publication should prove to be quite valuable in future research on complex continued fractions of this type. Jerry has participated by invitation in many research workshops and conferences in his field, including Boulder and Redstone, Colorado, Pitlochry and Aviemore, Scotland, Loen and Trondheim, Norway, and Campinas, Brazil.

Geraldine and Jerry have four children (two sons and two daughters) and nine grandchildren. Jerry retired on September 1, 1998, with the status of Professor Emeritus after thirty-eight years of service at MU. He is looking forward to enjoying mathematics, his family, and his other interests for a long time to come.

We wish the very best to Jerry in his future personal and professional endeavors.

Bruce C. Berndt
Fritz Gesztesy
Publications of L. J. Lange


$$\pi = 3 + \frac{1^2}{6 + \frac{3^2}{6 + \frac{5^2}{6 + \frac{7^2}{6 + \frac{9^2}{6 + \frac{11^2}{6 + \frac{13^2}{6 + \ldots}}}}}}}$$
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A Volume in Honor of L. J. Lange

Bruce C. Berndt and Fritz Gesztesy, Editors

This volume presents the contributions from the international conference held at the University of Missouri at Columbia, marking Professor Lange's 70th birthday and his retirement from the university. The principal purpose of the conference was to focus on continued fractions as a common interdisciplinary theme bridging gaps between a large number of fields—from pure mathematics to mathematical physics and approximation theory.

Evident in this work is the widespread influence of continued fractions in a broad range of areas of mathematics and physics, including number theory, elliptic functions, Padé approximations, orthogonal polynomials, moment problems, frequency analysis, and regularity properties of evolution equations. Different areas of current research are represented. The lectures at the conference and the contributions to this volume reflect the wide range of applicability of continued fractions in mathematics and the applied sciences.