The Functional and Harmonic Analysis of Wavelets and Frames

AMS Special Session on the Functional and Harmonic Analysis of Wavelets
January 13–14, 1999
San Antonio, Texas

Lawrence Wasson Baggett
David Royal Larson
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The proceedings of an AMS Special Session on The Functional and Harmonic Analysis of Wavelets, held at the Annual Meeting of the American Mathematical Society, San Antonio, TX, on January 13–14, 1999.

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Preface

In the past ten years, wavelets and frames have emerged as increasingly powerful tools of analysis on $\mathbb{R}^n$. Both wavelets and frames were initially studied primarily by using tools from classical Fourier analysis, but in recent years more abstract tools, e.g., from operator theory, from abstract harmonic analysis, from Von Neumann algebras, etc., have been introduced into these theories. We the editors have both spent much of our careers working in these more abstract areas of functional analysis, and we both view this application of modern analysis to wavelets and frames to both hold the promise of providing some deeper understanding of those subjects themselves as well as providing some fascinating new structures in so-called pure functional analysis. It was our hope that by organizing a special session in the “functional and harmonic” analysis of wavelets we would focus the attention of researchers on these newly-introduced tools. Happily, this appears to have occurred. Twenty-five speakers participated in our session; probing questions were asked; fruitful conversations were had; papers and preprints were exchanged; and strong interest was shown in our publishing a proceedings of the session.

Although our original session title only referred to wavelets, it is obvious to all that the notion of a frame is inextricably related to that of wavelets, and indeed many of the lectures in the session were devoted to frame theory. Hence, we have titled this volume *The Functional and Harmonic Analysis of Wavelets and Frames.*

It is our pleasure to thank all the participants in the special session as well as all their coauthors. We are also heavily indebted to Christine Thivierge, the Acquisitions Assistant for AMS, without whose guidance and encouragement this volume would have been much delayed and no doubt inferior.

Participants in the Special Session

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A. Aldroubi
R. Balan
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J. Courter
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The Functional and Harmonic Analysis of Wavelets and Frames
Lawrence Wasson Baggett and David Royal Larson, Editors

Over the past decade, wavelets and frames have emerged as increasingly powerful tools of analysis on $n$-dimension Euclidean space. Both wavelets and frames were studied initially by using classical Fourier analysis. However, in recent years more abstract tools have been introduced, for example, from operator theory, abstract harmonic analysis, von Neumann algebras, etc.

The editors of this volume organized a Special Session on the functional and harmonic analysis of wavelets at the San Antonio (TX) Joint Mathematics Meetings. The goal of the session was to focus research attention on these newly-introduced tools and to share the organizers’ view that this modern application holds the promise of providing some deeper understanding and fascinating new structures in pure functional analysis. This volume presents the fruitful results of the lively discussions that took place at the conference.