Nonlinear Wave Equations

A Conference in Honor of Walter A. Strauss
on the Occasion of His Sixtieth Birthday
May 2-3, 1998
Brown University

Yan Guo
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A Conference in Honor of Walter A. Strauss
on the Occasion of His Sixtieth Birthday
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Yan Guo
Editor
On May 2–3, 1998, a conference on Nonlinear Wave Equations was held at Brown University, Providence, RI, in honor of Walter A. Strauss on the occasion of his sixtieth birthday. This volume is a collection of refereed original papers and expository articles dedicated to him by his colleagues and friends.

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Preface

On May 2–3, 1998, a conference on Nonlinear Wave Equations was held at Brown University, Providence, to honor Walter A. Strauss on the occasion of his sixtieth birthday. Strauss is one of the pioneers in the field of nonlinear wave equations, and this volume of Contemporary Mathematics is a collection of refereed original papers and expository articles which are dedicated to him by his colleagues and friends. The articles range over a broad range of subjects, including scattering theory, dispersive waves, classical field theory, mathematical fluid dynamics, kinetic theory, stability theory and variational methods, to each of which he himself made important contributions.

Strauss practically invented the mathematical theory of scattering of nonlinear waves, particularly with his 1968 paper on the zero-mass case, and continuing with his joint 1972 work on the positive-mass case. This work eventually led to the remarkable conclusion, for large classes of nonlinear wave equations, that every finite-energy wave is a scattering state. Later on, in a long series of papers, he studied the scattering of linear waves, particularly the asymptotic decay of waves and the propagation of singularities outside a non-trapping obstacle. A high point of this work was the joint 1985 paper on moving obstacles and their scattering frequencies in electromagnetism.

Another line of Strauss' work dealt with the global existence of solutions of nonlinear wave equations, especially for dissipative equations and conservative equations. Using constrained variational methods, in 1977 he proved the existence of solitary wave solutions. This influential paper initiated a great deal of activity on nonlinear elliptic equations in unbounded space. He pioneered the analysis of a great variety of nonlinear wave equations, including those that occur in acoustics, quantum physics, nonlinear optics, gauge theory, fluids and plasmas. In this regard he was very influential in popularizing the use of sharp $L^p$ estimates in such wave equations.

In the 1980's and 1990's Strauss initiated a major effort to understand the stability and instability of solitary waves in conservative systems. The joint 1987 paper, with its sequel in 1990, set up general guidelines for such a theory. He and his students and collaborators have investigated stability for a large variety of particular systems. A particular focus of his more recent work has been on classical field theories, particularly on the kinetic theory of plasmas. He is a leader in the recent flowering of mathematical kinetic theory. The fundamental paper on the classical solutions of the Vlasov-Maxwell system is his joint 1986 paper. The major breakthrough in the analysis of instability of collisionless plasmas occurred with his joint 1995 paper.
This volume would not have been possible without the excellent contributions from all the authors. We are grateful to the referees for their constructive criticisms and suggestions. Special thanks go to Janice D'Amico and Dr. Luis Almeida for their help with the preparation of the TeX files. We are also grateful for the support and patience of the AMS, especially of Edward G. Dunne and Christine M. Thivierge.

We are profoundly honored that Tosio Kato contributed to this volume what is probably his last paper. As usual, he spent a lot of effort to make his contribution perfect in spite of trying circumstances. We are particularly sad that he suddenly passed away before seeing his paper in print.

Robert Glassey
Yan Guo
Jalal Shatah
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This volume presents original research papers and expository articles from the conference in honor of Walter A. Strauss's sixtieth birthday held at Brown University in Providence (RI). The book offers a collection of original papers and expository articles mainly devoted to the study of nonlinear wave equations. The articles cover a wide range of topics, including scattering theory, dispersive waves, classical field theory, mathematical fluid dynamics, kinetic theory, stability theory, and variational methods. The book offers a nice cross-section of current trends and research directions in the study of nonlinear wave equations and related topics.