

# CONTEMPORARY MATHEMATICS

266

## Singularities in Algebraic and Analytic Geometry

Caroline Grant Melles  
Ruth I. Michler  
Editors



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# Singularities in Algebraic and Analytic Geometry

# CONTEMPORARY MATHEMATICS

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American Mathematical Society  
Providence, Rhode Island

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1991 *Mathematics Subject Classification*. Primary 13B22, 14B05, 32S05, 32S20;  
Secondary 13F45, 14C17, 14E15, 14H30, 14J17, 32S40, 32S45, 32S50.

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### Library of Congress Cataloging-in-Publication Data

Singularities in algebraic and analytic geometry / Caroline Grant Melles, Ruth I. Michler, editors.  
p. cm. — (Contemporary mathematics, ISSN 0271-4132; 266)

Includes bibliographical references.

ISBN 0-8218-2005-2 (alk. paper)

1. Singularities (Mathematics)—Congresses. I. Melles, Caroline Grant, 1958– II. Michler, Ruth I. (Ruth Ingrid), 1967– III. Contemporary mathematics (American Mathematical Society); v. 266.

QA614.58.S5552 2000

516.3'5—dc21

00-064610

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10 9 8 7 6 5 4 3 2 1    05 04 03 02 01 00

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## Preface

This volume originated from a special session on singularities in algebraic and analytic geometry, which took place in 1999 at the national American Mathematical Society meeting in San Antonio, Texas. The year 1999 also marks the hundredth anniversary of the birth of the great algebraic geometer, Oscar Zariski. It seems especially fitting at this time to publish a collection of papers dedicated to the development of his ideas.

Zariski's influence on the study of singularities was evident both in the topics discussed and in the mathematical heritage of the presenters. Several talks related to fundamental work on resolution of singularities done by Zariski in the 1930's and 40's. Shreeram Abhyankar spoke about the history of the subject from a personal perspective, describing his voyage from India to the United States, how he met Zariski and became his student, and his subsequent work on resolution of singularities in characteristic  $p$ . Three of the contributors to this volume, Ban, Melles, and Roberts, were students of Zariski's students, and a fourth, McEwan, is a mathematical great-grandson.

At the time Zariski himself was a student in Rome, that city was the world's leading center of algebraic geometry. But a few years later, while at Johns Hopkins University in Baltimore, Zariski realized that the methods of the Italian school were lacking in rigor, and that to reach a deeper understanding of algebraic geometry he would need to use methods of commutative algebra to rebuild the entire foundation of the subject. Commutative algebra is the setting for three of the papers in this volume, those of Roberts, Vitulli, and Wiegand. Vitulli recounts how the concepts of weak normalization and weak subintegral closure arose from problems in classification of algebraic varieties, and proves a conjecture relating weak subintegral closures of ideals to Rees valuations. In a closely related paper, Roberts describes two equivalent ways to construct a universal weakly subintegral extension of a ring. Wiegand's paper on direct-sum decompositions of finitely generated modules is sprinkled with examples which illustrate how concrete algebraic methods can be.

Zariski's approach to the problem of resolution of singularities was much more algebraic than those of his predecessors. He gave algebraic proofs of resolution of singularities of surfaces and of local uniformization in all dimensions, using valuations of function fields. He introduced the modern notion of the blow-up of an ideal and recognized the importance of blow-ups of smooth centers. His work made possible later major developments, including the proof by his student Hironaka of the existence of resolution of singularities in characteristic zero in all dimensions. Cutkosky's article offers a modern view of valuations, highlighting the role valuation theory has played in algebraic geometry, outlining Zariski's proof of resolution

of surface singularities, and concluding with recent applications to the structure of morphisms.

Algorithmic approaches to resolution of singularities are featured in the papers of Ban and McEwan, and of Melles and Milman. Ban and McEwan prove simultaneous resolution of equisingular surface singularities using the algorithm given by Bierstone and Milman in their constructive and canonical proof of Hironaka's theorem. Melles and Milman describe combinatorial blow-ups and outline their use in resolving the singularities of locally toric varieties.

The papers of Némethi and Szilárd, on resolution graphs of surface singularities, also employ an algorithmic approach. Their examples and classification theorems illustrate the richness of the theory of surface singularities.

The theory of curves is another beautiful and fertile area. Abhyankar and Assi discuss recent work on Jacobians of meromorphic curves. McEwan examines spectral numbers of curve singularities, using Puiseux pairs.

Hypersurfaces form yet another class of spaces which are amenable to specialized techniques and provide a valuable testing ground for conjectures. Michler uses Gröbner basis calculations of Hochschild homology to investigate hypersurfaces with isolated singularities.

While Zariski was rebuilding the foundations of algebraic geometry, Severi and Todd, and then Chern, introduced characteristic classes as a way to study structures on complex algebraic varieties. Brasselet gives a brief history of the theory of characteristic classes of singular spaces and discusses some conjectures and open problems.

The editors are indebted to the contributors to the special session and to this volume, for their generosity and enthusiasm; to the referees, for their helpful suggestions, often under short deadlines; to the American Mathematical Society for its support of the special session and these proceedings; and to Christine M. Thivierge, acquisitions assistant at the AMS, for her editorial advice, patience, and encouragement.

In acknowledgment of his great contributions to algebraic geometry, this volume is dedicated to Oscar Zariski.

Caroline Grant Melles  
Ruth Michler  
May 2000



AMS SPECIAL SESSION ON  
SINGULARITIES IN ALGEBRAIC AND ANALYTIC GEOMETRY  
SAN ANTONIO, TEXAS

**Wednesday, January 13, 1999**

8:00 a.m. *Isolated Hypersurface Singularities with “Large” Torsion Module of Differentials*, Ruth I. Michler\*, University of North Texas.

8:30 a.m. *Berger’s Conjecture: A Problem in which Algebraic Geometry, Commutative Algebra, and Hochschild Homology Meet*, Sue Geller\*, Texas A & M University.

9:00 a.m. *Weighted Chern-Mather Classes and Milnor Classes*, Paolo Aluffi\*, Florida State University.

9:30 a.m. *Graph Manifolds for Algebraic Plane Curves*, Eriko Hironaka\*, Florida State University.

10:00 a.m. *Cohomology of the Link of a Surface Singularity in Positive Characteristic*, Aise J. de Jong\*, Massachusetts Institute of Technology.

2:15 p.m. *Canonical Resolution of Quasi-Ordinary Surface Singularities*, Lee J. McEwan\*, The Ohio State University at Mansfield, Chunsheng Ban, The Ohio State University at Mansfield.

2:45 p.m. *Deforming Artin Gorenstein Algebras, and Singularities*, Anthony A. Iarrobino, Jr.\*, Northeastern University.

3:15 p.m. *Valuations and Binomial Ideals*, Bernard H. Teissier\*, Université Paris 7, C.N.R.S., France.

4:15 p.m. *Euler Obstruction and Indices of Vector Fields*, Jean-Paul Brasselet\*, IML-CNRS, Luminy, France, Dũng Tráng Lê, Université de Provence Marseille, C.N.R.S., France, Jose Seade, Institute of Mathematics, National Autonomous University of Mexico.

5:15 p.m. *The Weak Subintegral Closure of a Monomial Ideal*, Les Reid, Southwest Missouri University, Marie A. Vitulli\*, University of Oregon.

**Thursday, January 14, 1999**

8:00 a.m. *An Explicit Construction of Complete Kähler Saper Metrics by Means of Desingularization*, Caroline Grant Melles\*, United States Naval Academy, Pierre Milman, University of Toronto.

8:30 a.m. *Stratification Conditions, Plane Sections and Equisingularity*, Terence J. Gaffney\*, Northeastern University.

9:00 a.m. *Resolution of Singularities and its History*, Shreeram S. Abhyankar\*, Purdue University.

10:00 a.m. *“Weakly” Elliptic Gorenstein Singularities of Surfaces*, Andras Nemethi\*, The Ohio State University.

11:00 a.m. *A Resolution Algorithm for a Special Class of Hypersurface Singularities*, András Némethi, The Ohio State University, Ágnes Szilárd\*, The Ohio State University.

11:30 a.m. *Minimal Models and Boundedness of Stable Varieties*, Kalle Karu\*, Boston University.

3:15 p.m. *Monomialization and Factorization of Morphisms*, Steven Dale Cutkosky\*, University of Missouri.

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\*indicates presenter

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## **Singularities in Algebraic and Analytic Geometry**

**Caroline Grant Melles and Ruth I. Michler, Editors**

This volume contains the proceedings of an AMS special session held at the 1999 Joint Mathematics Meetings in San Antonio. The participants were an international group of researchers studying singularities from algebraic and analytic viewpoints. The contributed papers contain original results as well as some expository and historical material. This volume is dedicated to Oscar Zariski, on the one hundredth anniversary of his birth.

Topics include the role of valuation theory in algebraic geometry with recent applications to the structure of morphisms; algorithmic approaches to resolution of equisingular surface singularities and locally toric varieties; weak subintegral closures of ideals and Rees valuations; constructions of universal weakly subintegral extensions of rings; direct-sum decompositions of finitely generated modules; construction and examples of resolution graphs of surface singularities; Jacobians of meromorphic curves; investigation of spectral numbers of curve singularities using Puiseux pairs; Gröbner basis calculations of Hochschild homology for hypersurfaces with isolated singularities; and the theory of characteristic classes of singular spaces—a brief history with conjectures and open problems.

ISBN 0-8218-2005-2



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