CONTEMPORARY MATHEMATICS

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Laminations and Foliations in Dynamics, Geometry and Topology

Proceedings of the Conference on Laminations and Foliations in Dynamics, Geometry and Topology
May 18–24, 1998
SUNY at Stony Brook

Mikhail Lyubich
John W. Milnor
Yair N. Minsky
Editors

American Mathematical Society

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Introduction

The concepts of lamination and foliation have recently gained fresh strength. They have enabled advances in long-standing problems in hyperbolic geometry, led to new concepts in topology, witnessed solution of classical problems on analytic differential equations, contributed to the renormalization theory of one-dimensional dynamical systems, and played a crucial role in the foundations of higher dimensional holomorphic dynamics.

In spite of their deep relations, these different topics have developed as distinct research fields, with little interaction between their practitioners. In an effort to overcome this separation and bring these diverse points of view closer together, we held a conference at Stony Brook in May of 1998. The conference brought together experts in the different aspects of lamination and foliation theory, as well as many young researchers and graduate students.

The Present Volume

All but the last of the papers in this volume are based on lectures given at the Stony Brook conference. The first two are based on minicourses from the conference:

- **Geodesic laminations,** by Francis Bonahon.
  One-dimensional geodesic laminations on hyperbolic surfaces arose in Thurston's work on Teichmüller spaces and hyperbolic 3-manifolds, where they play a central role. For example, in Teichmüller theory they allow one to describe asymptotic properties of degenerating hyperbolic metrics on the surface. In hyperbolic 3-manifolds they describe the bending of convex hulls and the asymptotic geometry of ends. This presentation sets out the basic theory of geodesic laminations, discussing both its topological aspects and the more analytical ones arising from consideration of transverse measures and distributions. Some applications are given to the geometry of convex-hull boundaries in hyperbolic 3-manifolds.

- **Laminations, Foliations and the Topology of 3-Manifolds,** by David Gabai.
  The notion of (two-dimensional) taut foliation in a 3-manifold, and the related notion of essential lamination, generalize in some ways the properties of incompressible surfaces and have been much developed and applied by Gabai, Hatcher, Oertel, Thurston and others. A recurring theme in this minicourse is the notion of "minimal position" of curves or surfaces with respect to a taut foliation or essential lamination, and the applications of these ideas to answering deep questions about knot theory, covering spaces and algebraic topology of 3-manifolds.

- **Dicritical singularities of holomorphic vector fields,** by Cesar Camacho.
  This paper discusses the local structure of singularities of holomorphic vector fields in $\mathbb{C}^2$ and of the associated foliations. It describes the resolution procedure and the monodromy group, and then gives criteria for existence of meromorphic and Liouvillian integrals, and introduces two new invariants of the singularity.
- **Dynamics of $\mathbb{P}^2$: examples,** by John Eric Fornaess and Nessim Sibony.

  The study of holomorphic dynamics in two or more complex dimensions is a relatively new but rapidly developing field. This paper explores the dynamics of holomorphic maps from the complex projective plane to itself, providing a number of surprising and instructive examples.

- **Rational laminations of complex polynomials,** by Jan Kiwi.
  If f is a monic polynomial map with connected Julia set, then the associated *rational lamination* is an equivalence relation on $\mathbb{Q}/\mathbb{Z}$, where two rational angles are equivalent if and only if the associated external rays land at a common point of the Julia set. This concept was perhaps first introduced by McMullen, although it is based on earlier work by Thurston, Douady and Hubbard. Two polynomials with the same rational lamination are sometimes said to be *combinatorially equivalent.* The present paper provides a big step towards understanding polynomials of higher degree by giving a complete characterization of those equivalence relations which can arise as rational laminations. The results are easy to state, but quite difficult to prove.

- **Actions of discrete groups on complex projective spaces,** by José Seade and Alberto Verjovsky.
  By definition, a higher dimensional "*Complex Kleinian Group*" is a discrete subgroup of $\mathrm{PSL}(n+1,\mathbb{C})$ which act on the projective space $P^n_{\mathbb{C}}$, $n > 1$ in such a way that the domain of discontinuity is non-empty. This paper provides a survey of the field. In particular, it discusses higher dimensional analogues of Fuchsian groups. Using twistor theory, it shows that every Kleinian group of conformal automorphisms of S^4 gives rise to a complex Kleinian group of automorphisms of $P^3_{\mathbb{C}}$.

- **Dynamics of singular holomorphic foliations on the complex projective plane,** by Saeed Zakeri.
  Any holomorphic foliation of the complex projective plane by curves, with only mild singularities, is induced by a polynomial vector field on the plane. This is an outline of the theory, including discussion of concepts of 'degree', of monodromy, density of leaves, and ergodicity, as well as the possible existence of foliations with a non-trivial minimal set.

The Stony Brook Conference

A number of the lectures at the conference are not represented in this volume. First there were two further minicourses:

- **Dynamics and currents in $\mathbb{C}^2$,** by E. Bedford and J. Smillie.
  Classically, laminations arise in the theory of hyperbolic dynamical systems as families of stable and unstable manifolds. There are natural "Ruelle-Sullivan currents" associated with these laminations. Remarkably, in two-dimensional holomorphic dynamics (iteration theory of polynomial diffeomorphisms of $\mathbb{C}^2$), currents come first as distributional $\partial\bar{\partial}$-derivatives of certain pluripotential Green's functions. These can then be interpreted as geometric currents supported on (1-complex-dimensional) laminations, which provide deep insight into the topological dynamics of the map.

  [For much of this material, see for example the series of papers "*Polynomial diffeomorphisms of* $\mathbb{C}^2$" by Bedford and Smillie, in Invent. Math. **103** (1991) and **112** (1993) (joint with Lyubich), J. Amer. Math. Soc. **4** (1991), Math. Ann. **294** (1992), J. Geom. Anal. **8** (1998), Ann. Math. **148** (1998), and Ann. Sci. Éc. Norm. Sup. **32** (1999).]

- **Riemann surface laminations: uniformization and meromorphic functions,** by É. Ghys.
  This minicourse focused on two foundational problems in the theory of Riemann surface laminations: the problem of embedding a Riemann surface lamination into a projective space, and the uniformization problem for Riemann surface laminations. These are motivated by the classical theory of Riemann surfaces, the Poincaré-Bendixon theory, and a conjecture of Camacho. The lectures developed a conceptual background for the discussion (foliated cycles, harmonic currents, divisors...) and presented many fascinating examples (e.g., an example of a non-flat lamination with all leaves parabolic) and several nice results (e.g., some necessary conditions for the embedding into a projective space) in this direction.

  [An exposition of these ideas can be found in "*Laminations par surfaces de Riemann*", Panoramas et Synthèses **8** (1999), 49–95.]

In addition to the minicourses, there were a number of one-hour talks on current research topics:

- C. Camacho. *Complex foliations near a singularity.*
- A. Connes. *The Riemann flow and the Zeros of Zeta.*
- Ya. Eliashberg. *Foliations and contact structures on 3-manifolds.*
- S. Fenley. *Foliations with good geometry.*
- J.-E. Fornaess. *Remarks on dynamics on* $\mathbb{P}^2$.
- X. Gomez-Mont. *The attractor of a Riccati equation and the foliated geodesic flow.*
- A. Hatcher. *Kontsevich's conjecture on diffeomorphism groups of 3-manifolds.*
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- S. Kerkhoff. *Seifert fibered spaces in the orbifold theorem.*
- J. Kiwi. *Rational laminations of complex polynomials.*

- L. Mosher. *Laminations and solvable groups.*
- D. Sullivan. *Some remarks on quantum topology.*
- A. Verjovsky. *Complex Kleinian groups in higher dimensions.*

Finally, two parallel evening sessions gave many active researchers an opportunity to present their recent results in the field.

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Mikhail Lyubich, John Milnor, Yair Minsky
Stony Brook, July 2000

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