CONTEMPORARY MATHEMATICS

280

Structured Matrices in Mathematics, Computer Science, and Engineering I

Proceedings of an AMS-IMS-SIAM
Joint Summer Research Conference
University of Colorado, Boulder
June 27–July 1, 1999

Vadim Olshevsky
Editor

American Mathematical Society

http://dx.doi.org/10.1090/conm/280

Selected Titles in This Series

280 **Vadim Olshevsky, Editor,** Structured matrices in mathematics, computer science, and engineering I, 2001

279 **Alejandro Adem, Gunnar Carlsson, and Ralph Cohen, Editors,** Topology, geometry, and algebra: Interactions and new directions, 2001

278 **Eric Todd Quinto, Leon Ehrenpreis, Adel Faridani, Fulton Gonzalez, and Eric Grinberg, Editors,** Radon transforms and tomography, 2001

277 **Luca Capogna and Loredana Lanzani, Editors,** Harmonic analysis and boundary value problems, 2001

276 **Emma Previato, Editor,** Advances in algebraic geometry motivated by physics, 2001

275 **Alfred G. Noël, Earl Barnes, and Sonya A. F. Stephens, Editors,** Council for African American researchers in the mathematical sciences: Volume III, 2001

274 **Ken-ichi Maruyama and John W. Rutter, Editors,** Groups of homotopy self-equivalences and related topics, 2001

273 **A. V. Kelarev, R. Göbel, K. M. Rangaswamy, P. Schultz, and C. Vinsonhaler, Editors,** Abelian groups, rings and modules, 2001

272 **Eva Bayer-Fluckiger, David Lewis, and Andrew Ranicki, Editors,** Quadratic forms and their applications, 2000

271 **J. P. C. Greenlees, Robert R. Bruner, and Nicholas Kuhn, Editors,** Homotopy methods in algebraic topology, 2001

270 **Jan Denef, Leonard Lipschitz, Thanases Pheidas, and Jan Van Geel, Editors,** Hilbert's tenth problem: Relations with arithmetic and algebraic geometry, 2000

269 **Mikhail Lyubich, John W. Milnor, and Yair N. Minsky, Editors,** Laminations and foliations in dynamics, geometry and topology, 2001

268 **Robert Gulliver, Walter Littman, and Roberto Triggiani, Editors,** Differential geometric methods in the control of partial differential equations, 2000

267 **Nicolás Andruskiewitsch, Walter Ricardo Ferrer Santos, and Hans-Jürgen Schneider, Editors,** New trends in Hopf algebra theory, 2000

266 **Caroline Grant Melles and Ruth I. Michler, Editors,** Singularities in algebraic and analytic geometry, 2000

265 **Dominique Arlettaz and Kathryn Hess, Editors,** Une dḝgustation topologique: Homotopy theory in the Swiss Alps, 2000

264 **Kai Yuen Chan, Alexander A. Mikhalev, Man-Keung Siu, Jie-Tai Yu, and Efim I. Zelmanov, Editors,** Combinatorial and computational algebra, 2000

263 **Yan Guo, Editor,** Nonlinear wave equations, 2000

262 **Paul Igodt, Herbert Abels, Yves Félix, and Fritz Grunewald, Editors,** Crystallographic groups and their generalizations, 2000

261 **Gregory Budzban, Philip Feinsilver, and Arun Mukherjea, Editors,** Probability on algebraic structures, 2000

260 **Salvador Pérez-Esteva and Carlos Villegas-Blas, Editors,** First summer school in analysis and mathematical physics: Quantization, the Segal-Bargmann transform and semiclassical analysis, 2000

259 **D. V. Huynh, S. K. Jain, and S. R. López-Permouth, Editors,** Algebra and its applications, 2000

258 **Karsten Grove, Ib Henning Madsen, and Erik Kjær Pedersen, Editors,** Geometry and topology: Aarhus, 2000

257 **Peter A. Cholak, Steffen Lempp, Manuel Lerman, and Richard A. Shore, Editors,** Computability theory and its applications: Current trends and open problems, 2000

256 **Irwin Kra and Bernard Maskit, Editors,** In the tradition of Ahlfors and Bers: Proceedings of the first Ahlfors-Bers colloquium, 2000

(*Continued in the back of this publication*)

Structured Matrices in Mathematics, Computer Science, and Engineering I

CONTEMPORARY MATHEMATICS

280

Structured Matrices in Mathematics, Computer Science, and Engineering I

Proceedings of an AMS-IMS-SIAM
Joint Summer Research Conference
University of Colorado, Boulder
June 27–July 1, 1999

Vadim Olshevsky
Editor

American Mathematical Society
Providence, Rhode Island

Editorial Board
Dennis DeTurck, managing editor
Andreas Blass Andy R. Magid Michael Vogelius

This volume contains the proceedings of an AMS-IMS-SIAM Joint Summer Research Conference held at the University of Colorado, Boulder, Colorado on June 27–July 1, 1999, with support from the National Science Foundation, grant DMS-9618514.

2000 *Mathematics Subject Classification.* Primary 15–XX, 47–XX, 65–XX, 93–XX.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Library of Congress Cataloging-in-Publication Data

Structured matrices in mathematics, computer science, and engineering : proceedings of an AMS-IMS-SIAM joint summer research conference, University of Colorado, Boulder, June 27–July 1, 1999 / Vadim Olshevsky, editor.
p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 280–281)
Includes bibliographical references.
ISBN 0-8218-1921-6 (v. 1 : alk. paper)–ISBN 0-8218-2092-3 (v. 2 : alk. paper)
1. Matrices—Congresses. I. Olshevsky, Vadim, 1961– II. Contemporary mathematics (American Mathematical Society) ; v. 280–281.

QA188 .S764 2001
512.9′434—dc21 2001041241

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Assistant to the Publisher, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to `reprint-permission@ams.org`.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2001 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights
except those granted to the United States Government.

Printed in the United States of America.

♾ The paper used in this book is acid-free and falls within the guidelines
established to ensure permanence and durability.
Visit the AMS home page at URL: `http://www.ams.org/`

10 9 8 7 6 5 4 3 2 1 06 05 04 03 02 01

Contents

Structured Matrices in Mathematics, Computer Science, and Engineering I

Part I. Interpolation and Approximation

Part II. System Theory, Signal and Image Processing

Part III. Control Theory

Part IV. Spectral Properties. Conditioning

Structured Matrices in Mathematics, Computer Science, and Engineering II

Part V. Fast Algorithms

Part VI. Numerical Issues

Part VII. Iterative Methods. Preconditioners

Part VIII. Linear Algebra and Various Applications

Foreword

Many important problems in applied sciences, mathematics, and engineering can be reduced to matrix problems. Moreover, various applications often introduce a special structure into the corresponding matrices, so that their entries can be described by a certain compact formula. Among classical examples are Toeplitz matrices $[a_{i-j}]$, Hankel matrices $[a_{i+j}]$, Toeplitz-plus-Hankel matrices, Vandermonde matrices $[a_i^{j-1}]$, Cauchy matrices $[\frac{1}{a_i-b_j}]$, Pick matrices $[\frac{1-a_i a_j^*}{b_i+b_j^*}]$, and also Bezoutians, controllability and observability matrices and others. Though standard linear algebra methods are, of course, readily available, there are several reasons why they can be unattractive in many instances. Along with just the desire to find elegant structure-exploiting solutions, there are also practical computational considerations such as the storage limitations, the need in reducing computational complexity as well as in obtaining a better numerical accuracy. In many cases these goals can be achieved by solving the underlying problem in terms of only $O(n)$ of parameters defining a structured $n \times n$ matrix via a compact formula as in the above examples.

Structured matrices have been under close study for a long time, and in quite diverse (and seemingly unrelated) areas. Typically, not only an area of application gives rise to certain patterns of structure, but it often provides a technique to solve the associated matrix problems. As an illustration of this principle we mention the classical interpolation problems of Caratheodory-Toeplitz and of Nevanlinna-Pick. Not only are they related to positive definite Toeplitz and Pick matrices, resp., but the recursive algorithms of Schur and Nevanlinna for computing their solutions admit nice matrix interpretations. In fact, these classical interpolation algorithms can be understood as efficient structure-exploiting ways to compute the Cholesky decompositions for these matrices. As was mentioned above, structured matrices were studied from different points of view, in mathematics, computer science and engineering. For example, the same Toeplitz and Pick matrices were closely looked at using the methods of reproducing kernel Hilbert spaces, lifting-of-commutants, state-space methods, as well as the methods of system theory and signal processing, network theory, linear prediction, to mention just a few mathematical and engineering fields. Interestingly, in the latter a physical intuition often provides deep insights into structured matrix problems. An interplay between the techniques of engineers and mathematicians is reflected in these volumes. There are several other areas providing their own applications and their own languages to attack structured matrix problems. It can be quite difficult to survey all such connections; in fact, browsing through the papers of these volumes can give a flavor of the plethora of different techniques and approaches. It appears that the theory of structured matrices is positioned to bridge the gaps between these diverse areas.

Significant progress has been recently made in studying relevant numerical issues. It was quite well-understood for a long time that structure can be exploited to speed-up computations and to design fast algorithms. However, many of those suffered from the loss of numerical accuracy. In the past few years a number of algorithms blending speed and accuracy has been developed. This progress is fully reflected in these volumes.

Though structured matrices have been under close study for a long time, in the past decade they have enjoyed a significant growth in popularity. One reason for this is in that the theory of structured matrices is poised to bridge diverse applications in the sciences and engineering, deep mathematical theories and computational and numerical issues. Hence, it is not surprising that the number of researchers in our scientific community is rapidly increasing. Special sessions and minisymposia devoted to structured matrices were included in the programs of various general mathematical and engineering conferences, among which we mention various SIAM meetings, ILAS, IWOTA, SPIE, and MTNS. Moreover, in the past few years several special international conferences focusing solely on different aspects of structured matrices were held in Santa Barbara (USA, Aug. 1996), Cortona (Italy, Sept. 1996), Boulder (USA, July 1999), Chemnitz(Germany, Jan. 2000), Cortona (Italy, Sep. 2000), and we are looking forward to the next meeting in South Hadley (USA, Aug. 2001).

These meetings, especially the one in Boulder (July 1999), brought together quite diverse audiences of participants, many of which have never actually met earlier in the framework of such a comprehensive cross-disciplinary conference. In fact, it was a unique "cross-fertilization" atmosphere of the Boulder meeting that suggested the idea to pursue this publishing project. The detailed table of contents will provide a general idea of these volumes. Thirty eight papers devoted to the different aspects of the theory of structured matrices and using different techniques are collected under one cover. We hope that the reader will enjoy a plethora of different problems, different focuses, and different methods that all contribute to one unified theory of structured matrices.

Part I is devoted to a connection of structured matrices to several problems in interpolation and approximation. In the first paper the reader will see a natural connection of our subject to reproducing kernels. Other papers emphasize algorithmical aspects of tangential interpolation problems, exploit the maximum-volume concept, and present relations to Pade approximations and to the extended Euclidean algorithm.

Part II provides the perspective of engineers and presents their insights into the subject. It starts with a paper on systems on low Hankel rank. The reader will notice that the other parts of these volumes contain several other papers that discuss some of the related issues but from different points of view. Part 2 also contains an application of the "language of signal flow graphs" to study structured matrices, an exposition of recent advances in tensor analysis, and several applications.

Part III is dedicated to recent relevant developments in control theory.

Part IV contains several papers discussing spectral properties of Toeplitz, block Toeplitz, Pick and Hankel matrices.

PART V presents several fast algorithms for various classes of structured matrices.

Part VI provides a snapshot of the current state-of-art in numerical issues related to structured matrices. It starts with a paper containing a "quite heavy" error analysis leading to the first provably stable variant of the unitary Hessenberg QR algorithm. The other papers cover stability issues of fast algorithms and inversion formulas.

Part VII contains two papers devoted to preconditioners design.

Finally, part VIII contains a discussion of the concept of approximate displacement rank, new inversion formulas for structured matrices, a survey on completion problems and several related topics.

Vadim Olshevsky

Department of Mathematics and Statistics
Georgia State University
Atlanta, GA 30303, USA

Selected Titles in This Series

(*Continued from the front of this publication*)

255 **Jerry Bona, Katarzyna Saxton, and Ralph Saxton, Editors,** Nonlinear PDE's, dynamics and continuum physics, 2000

254 **Mourad E. H. Ismail and Dennis W. Stanton, Editors,** q-series from a contemporary perspective, 2000

253 **Charles N. Delzell and James J. Madden, Editors,** Real algebraic geometry and ordered structures, 2000

252 **Nathaniel Dean, Cassandra M. McZeal, and Pamela J. Williams, Editors,** African Americans in Mathematics II, 1999

251 **Eric L. Grinberg, Shiferaw Berhanu, Marvin I. Knopp, Gerardo A. Mendoza, and Eric Todd Quinto, Editors,** Analysis, geometry, number theory: The Mathematics of Leon Ehrenpreis, 2000

250 **Robert H. Gilman, Editor,** Groups, languages and geometry, 1999

249 **Myung-Hwan Kim, John S. Hsia, Yoshiyuki Kitaoka, and Rainer Schulze-Pillot, Editors,** Integral quadratic forms and lattices, 1999

248 **Naihuan Jing and Kailash C. Misra, Editors,** Recent developments in quantum affine algebras and related topics, 1999

247 **Lawrence Wasson Baggett and David Royal Larson, Editors,** The functional and harmonic analysis of wavelets and frames, 1999

246 **Marcy Barge and Krystyna Kuperberg, Editors,** Geometry and topology in dynamics, 1999

245 **Michael D. Fried, Editor,** Applications of curves over finite fields, 1999

244 **Leovigildo Alonso Tarrío, Ana Jeremías López, and Joseph Lipman,** Studies in duality on noetherian formal schemes and non-noetherian ordinary schemes, 1999

243 **Tsit Yuan Lam and Andy R. Magid, Editors,** Algebra, K-theory, groups, and education, 1999

242 **Bernhelm Booss-Bavnbek and Krzysztof Wojciechowski, Editors,** Geometric aspects of partial differential equations, 1999

241 **Piotr Pragacz, Michał Szurek, and Jarosław Wiśniewski, Editors,** Algebraic geometry: Hirzebruch 70, 1999

240 **Angel Carocca, Víctor González-Aguilera, and Rubí E. Rodríguez, Editors,** Complex geometry of groups, 1999

239 **Jean-Pierre Meyer, Jack Morava, and W. Stephen Wilson, Editors,** Homotopy invariant algebraic structures, 1999

238 **Gui-Qiang Chen and Emmanuele DiBenedetto, Editors,** Nonlinear partial differential equations, 1999

237 **Thomas Branson, Editor,** Spectral problems in geometry and arithmetic, 1999

236 **Bruce C. Berndt and Fritz Gesztesy, Editors,** Continued fractions: From analytic number theory to constructive approximation, 1999

235 **Walter A. Carnielli and Itala M. L. D'Ottaviano, Editors,** Advances in contemporary logic and computer science, 1999

234 **Theodore P. Hill and Christian Houdré, Editors,** Advances in stochastic inequalities, 1999

233 **Hanna Nencka, Editor,** Low dimensional topology, 1999

232 **Krzysztof Jarosz, Editor,** Function spaces, 1999

231 **Michael Farber, Wolfgang Lück, and Shmuel Weinberger, Editors,** Tel Aviv topology conference: Rothenberg Festschrift, 1999

230 **Ezra Getzler and Mikhail Kapranov, Editors,** Higher category theory, 1998

For a complete list of titles in this series, visit the
AMS Bookstore at **www.ams.org/bookstore/**.

Structured Matrices in Mathematics, Computer Science, and Engineering I

Vadim Olshevsky, Editor

Many important problems in applied sciences, mathematics, and engineering can be reduced to matrix problems. Moreover, various applications often introduce a special structure into the corresponding matrices, so that their entries can be described by a certain compact formula. Classic examples include Toeplitz matrices, Hankel matrices, Vandermonde matrices, Cauchy matrices, Pick matrices, Bezoutians, controllability and observability matrices, and others. Exploiting these and the more general structures often allows us to obtain elegant solutions to mathematical problems as well as to design more efficient practical algorithms for a variety of applied engineering problems.

Structured matrices have been under close study for a long time and in quite diverse (and seemingly unrelated) areas, for example, mathematics, computer science, and engineering. Considerable progress has recently been made in all these areas, and especially in studying the relevant numerical and computational issues. In the past few years, a number of practical algorithms blending speed and accuracy have been developed. This significant growth is fully reflected in these volumes, which collect 38 papers devoted to the numerous aspects of the topic.

The collection of the contributions to these volumes offers a flavor of the plethora of different approaches to attack structured matrix problems. The reader will find that the theory of structured matrices is positioned to bridge diverse applications in the sciences and engineering, deep mathematical theories, as well as computational and numerical issues. The presentation fully illustrates the fact that the techniques of engineers, mathematicians, and numerical analysts nicely complement each other, and they all contribute to one unified theory of structured matrices.

The book is published in two volumes. The first contains articles on interpolation, system theory, signal and image processing, control theory, and spectral theory. Articles in the second volume are devoted to fast algorithms, numerical and iterative methods, and various applications.

ISBN 0-8218-1921-6

9 780821 819210

CONM/280

AMS *on the* Web
www.ams.org