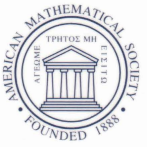
CONTEMPORARY MATHEMATICS

302

Logic and Algebra

Edited by Yi Zhang

with a Preface by Oleg Belegradek



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American Mathematical Society
Providence, Rhode Island

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This volume was inspired by a series of lectures in logic and algebra given at three conferences and weekly seminars at Istanbul Bilgi University during the 1999–2000 and 2000–2001 academic years.

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Preface

This volume was inspired by the series of lectures in logic and algebra given at the three conferences and weekly seminars in Istanbul Bilgi University over the 1999/00 and 2000/01 academic years. Most contributors to the book have spoken at the meetings, and some of the contributions are expanded versions of the talks presented there. The main topics of the volume are model theory with its applications to algebra, and set theory.

The papers in this book written by Altinel, Baldwin, Grossberg, Hodges, Hyttinen, Lessmann and Zilber deal with various questions of model theory whose common feature is that all of them concerns stability theory or grew up of it.

Altinel's paper is a survey of the state of the art of classification of infinite simple groups of finite Morley rank. Baldwin oulines a modern exposition of the basic theory of forking and multiplicity, first establishing results which hold for all theories, then restricting to simple theories, and finally to stable theories. Grossberg discusses the basics and perspectives of the classification theory for abstract elementary classes. Hodges' paper, where he classifies relatively categorical theories of linear orderings with distinguished subordering, is intended to serve as an introduction to relative categoricity. Hyttinen introduces and studies a notion of a group acting on a geometry, which is a generalization of objects known in geometric stability theory. Lessmann's paper is a contribution to homogeneous model theory, where he gives a condition for existence of arbitrary large homogeneous models omitting prescribed types, and proves an analog of the Baldwin–Lachlan theorem for atomic models. Zilber constructs an ω-stable complete theory of "pseudo-analytic" structures on algebraically closed fields; this result is a part of his program of model-theoretic study of analytic structures by means of including Hrushovski's method into the analytic context.

Belegradek studies a certain class of ordered abelian groups tractable from the model-theoretic point of view. Bovykin and Kaye give a survey of results on the possible order-types of non-standard models of Peano Arithmetic. Di Nasso and Forti investigate the order-types of the non-standard real line and its parts. The paper of Di Nasso and Zhang provides an introduction to non-standard analysis in purely algebraic terms, and, as an application, a non-standard proof of a characterization theorem for compact subsets of $\mathrm{Sym}(\omega)$. Tolstykh studies the expressive power of the first order logic for the automorphism groups of infinitely generated free nilpotent groups.

Hamkins surveys results on the automorphism towers of groups, from the classical algebraic facts to the recent results with a set-theoretic flavor due to Thomas and himself. The papers of Blass and Brendle are concerned with cardinal characteristics of the continuum. Blass considers some notions around the dominating,

splitting and ultrafilter numbers. Brendle provides an introduction to Shelah's recent theory of iterations along templates developed to prove the consistency of $\mathfrak{d} < \mathfrak{a}$. Stavi and Väänanen study certain reflection principles for the set of all sets of hereditary cardinality less that 2^{ω}.

The logic conferences and seminars which inspired the book were sponsored by Istanbul Bilgi University and partially by NATO via some TÜBITAK grants; without this financial support the activities could not take place. Also, these activities would not have been the success without tireless work of Yi Zhang on all aspects of the organization of the conferences and seminars.

Oleg Belegradek
Istanbul Bilgi University
May, 2002

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This volume outlines current developments in model theory and combinatorial set theory and presents state-of-the-art research. Well-known researchers report on their work in model theory and set theory with applications to algebra.

The papers of J. Brendle and A. Blass present one of the most interesting areas of set theory. Brendle gives a very detailed and readable account of Shelah's solution for the long-standing problem of $\mathrm{Con}(\mathfrak{d} < \mathfrak{a})$. It could be used in an advanced graduate seminar on set theory.

Papers by T. Altinel, J. T. Baldwin, R. Grossberg, W. Hodges, T. Hyttinen, O. Lessmann, and B. Zilber deal with questions of model theory from the viewpoint of stability theory. Here, Zilber constructs an ω-stable complete theory of "pseudo-analytic" structures on algebraically closed fields. This result is part of his program of the model-theoretic study of analytic structures by including Hrushovski's method in the analytic context.

The book presents this and further developments in model theory. It is geared toward advanced graduate students and researchers interested in logic and foundations, algebra, and algebraic geometry.

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