Orbifolds in Mathematics and Physics

Proceedings of a Conference on Mathematical Aspects of Orbifold String Theory
May 4–8, 2001
University of Wisconsin, Madison, Wisconsin

Alejandro Adem
Jack Morava
Yongbin Ruan
Editors
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Preface

In May 2001 the Department of Mathematics at the University of Wisconsin, hosted a conference in Madison on Mathematical Aspects of Orbifold String Theory. The meeting brought together senior workers from mathematical physics, algebraic geometry, algebraic topology, symplectic geometry and representation theory, as well as a very strong group of young researchers in these fields, including a number of graduate students.


The subject has a long prehistory, going back to work of Thurston and Haefliger, with roots in the theory of manifolds, group actions, and foliations, but the recent explosion of activity has been powered by applications to moduli problems and quantum field theory. The current state of research is impressive for its rapid growth and hybrid vigor.

The meeting accomplished its stated goal of creating interactions between a broad range of mathematicians and physicists focused on problems related to orbifolds. Topics such as stacks, vertex operator algebras, branes, groupoids, K-theory, quantum cohomology were all discussed in this context, and there was a dynamic and fruitful exchange of knowledge and methodology. This mix of interest among fields and the youth and enthusiasm of the audience made for a remarkable conference.

This volume contains 14 refereed papers on topics related to orbifolds. They represent the diverse nature of the mathematics and physics represented at the meeting, and include papers by distinguished physicists, algebraists, topologists and algebraic geometers. All the papers were carefully refereed. As one can readily see this volume contains a huge quantity of information on a subject which until now has not been adequately covered in the literature, and it is our expectation that our proceedings volume will become a showcase for the current state of the art in orbifold theory.

We would like to thank the National Science Foundation as well as the UW-Madison Math Department for their generous financial support. We would also like
to thank Peggy Conklin for her help as an organizer. Finally we owe a big debt of gratitude to Diane Reppert for her fabulous help in assembling this proceedings volume.

Alejandro Adem
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This book publishes papers originally presented at a conference on the Mathematical Aspects of Orbifold String Theory, hosted by the University of Wisconsin-Madison. The recent explosion of activity on the topic has been powered by applications of orbifolds to moduli problems and quantum field theory. The present volume contains information not fully covered in the published literature, and presents an interdisciplinary look at orbifold problems. Topics such as stacks, vertex operator algebras, branes, groupoids, K-theory and quantum cohomology are discussed. The book reflects the thinking of distinguished investigators working in the areas of mathematical physics, algebraic geometry, algebraic topology, symplectic geometry and representation theory. Advanced graduate students and researchers working in these areas, as well as those interested in connections between mathematical subject areas, will find this book of interest.