CONTEMPORARY MATHEMATICS

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2001 AMS-IMS-SIAM Joint Summer Research Conference on Lusternik-Schnirelmann Category in the New Millennium
July 29–August 2, 2001
Mount Holyoke College, South Hadley, Massachusetts

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http://dx.doi.org/10.1090/conm/316

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The 2001 AMS-IMS-SIAM Joint Summer Research Conference on "Lusternik-Schnirelmann Category in the New Millennium" was held at Mount Holyoke College, South Hadley, Massachusetts, July 29–August 2, 2001, with support from the National Science Foundation, grant DMS 9973450.

2000 *Mathematics Subject Classification.* Primary 55M30, 55P45, 55P60, 55P62, 55P91, 55Q25, 57M99, 57R30, 58E05.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Library of Congress Cataloging-in-Publication Data

AMS-IMS-SIAM Joint Summer Research Conference on Lusternik-Schnirelmann Category in the New Millennium (2001: Mount Holyoke College)

Lusternik-Schnirelmann Category and related topics: 2001 AMS-IMS-SIAM Joint Summer Research Conference on Lusternik-Schnirelmann Category in the New Millennium, Mount Holyoke College, South Hadley, Massachusetts, July 29–August 2, 2001 / O. Cornea... [et al.], editors.

p. cm. — (Contemporary Mathematics, ISSN 0271-4132; 316)

ISBN 0-8218-2800-2 (alk. paper)

1. Lusternik-Schnirelmann category—Congresses. 2. Algebraic topology—Congresses. I. Cornea, O. (Octavian), 1966– . II. Title. III. Contemporary mathematics (American Mathematical Society); v. 316.

QA612.A74 2001
514′.24—dc21 2002038243

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Preface

This collection of articles is the proceedings volume for the American Mathematical Society's Summer Research Conference, *Lusternik-Schnirelmann Category in the New Millennium*, held 29th July–2nd August 2001 on the campus of Mount Holyoke College in Massachusetts. The conference, one of seven joint AMS-IMS-SIAM summer research conferences in the mathematical sciences held that summer, attracted an international group of 37 participants that included many of the leading practitioners in the field.

Lusternik-Schnirelmann category (LS-category) is an integer that can be associated to a topological space. It is an invariant of the homotopy type of the space that gives a numerical measure of the complexity of the space. In particular, it indicates the complexity of possible dynamics on a smooth manifold, by providing a lower bound on the number of critical points of any smooth function on the manifold. The survey article by HILTON in this volume recalls the basic definitions and those results that first brought category to the attention of topologists.

While LS-category is classical in origin, the subject has recently enjoyed a renaissance. The latest developments include work in the areas of homotopy theory, dynamical systems, and symplectic topology. One interesting aspect of this recent activity is the way in which it has made significant links connecting these areas.

Many of the new developments have occurred in the homotopy-theoretic branch of the subject. The composition of the articles in this volume reflects this fact. Of the fifteen contributed articles, nine are primarily homotopy-theoretic. The survey article by HILTON gives a resumé of those homotopy-theoretic results known before the surge of recent activity. While several of the contributed articles continue in this classical vein, practically all of them are influenced in one way or another by more recent ideas. Certainly all of them represent non-classical points of view on LS-category.

Broadly speaking, the main forces responsible for spurring new interest in LS-category among homotopy theorists have been developments concerning the Ganea conjecture (that $\mathrm{cat}(X \times S^n) = \mathrm{cat}(X) + 1$) on the one hand and rational homotopy theory on the other. Concerning the Ganea conjecture, there are two ideas that have played a particularly central role in this work. One is the notion of *category weight* and the other is the notion of *Hopf invariant*, in both a classical and an extended sense. The articles of STROM and OPREA-RUDYAK are concerned with category weight, those of DULA and MARCUM with Hopf invariants. The articles by CUVILLIEZ-FÉLIX and LUPTON are concerned with rational homotopy theory proper, and those of COSTOYA-RAMOS, GHIENNE, and HUBBUCK-IWASE with more general localization and completion. Finally, among the primarily homotopy-theoretic articles, those of ARKOWITZ-STANLEY-STROM, MARCUM, and STROM

are concerned with extensions to a categorical setting of the notions of LS-category, Hopf invariants, and category weight, respectively.

In addition to these primarily homotopy-theoretic articles, three more give applications to other fields from a homotopy point of view. The articles of COLMAN, OPREA, and OPREA-RUDYAK give applications to finite group actions, torus actions and 3-manifolds, respectively.

We have mentioned that Hopf invariants have played a major role in recent homotopy-theoretic developments. Somewhat surprisingly perhaps, these have been connected with dynamical systems as well. Indeed, there is a whole branch consisting of a homotopical approach to dynamics, essentially developing from Morse theory and the work of Conley. This work linking the homotopical development of LS-category to dynamics has also spilled over into the world of symplectic topology, where category and related invariants have proved to be useful tools in investigating subjects such as Hamiltonian circle actions and the Arnold conjecture (for fixed points *and* Lagrangian intersections). The dynamical viewpoint is represented in this volume by the article of FARBER. Several of the references in his article give a starting point from which to delve into this branch. Also in the area of dynamical systems, but in a decidedly more classical vein, the article by GAVRILA gives a version of the original Lusternik-Schnirelmann theorem for a closely-related invariant.

Finally, the article by COLMAN-HURDER represents an area in which ideas connected with LS-category have only recently been recognized as important, namely foliations. The references in that article provide many sources for this promising area of development. The article, and indeed the appearance of LS-category in foliations, illustrates very well the kind of cross-fertilization that we hoped to foster at the conference.

One word about notation: The original definition of LS-category, as given in the article of HILTON, would yield the LS-category of a sphere as 2. In homotopy theory, however, it is usual to adjust the definition by 1 in such a way that the sphere has LS-category equal to 1. The articles by HILTON, COLMAN, COLMAN-HURDER, FARBER, and GAVRILA adopt the former convention, while all the other articles in this volume adopt the latter.

We would like to thank the AMS on several counts. First, we must thank it in a global way for its financial support of the Summer Research Conferences. In a local way, we thank it for its financial and administrative support for our conference. In particular, our on-site administrative staff person was Donna Salter, and it can safely be ventured that the success of the conference was due in large measure to her organizational skills. Subsequently, the AMS publications department has been very encouraging and supportive throughout the preparation of this volume. We would like especially to thank Christine Thivierge for her guidance at each stage.

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This collection is the proceedings volume for the American Mathematical Society's Summer Research Conference, Lusternik-Schnirelmann Category, held in 2001 at Mount Holyoke College in Massachusetts. The conference and its contributions here represent an international group of the leading practitioners in the field.

With a surge of recent activity, exciting advances have been made in this field, including the resolution of several long-standing conjectures. Lusternik-Schnirelmann category is a numerical homotopy invariant that also provides a lower bound for the number of critical points of a smooth function on a manifold. The study of this invariant, together with related notions, forms a subject lying on the boundary between homotopy theory and critical point theory.

These articles cover a wide range of topics for research mathematicians and graduate students. Some focus on concrete computations and applications while others look at more abstract extensions of the fundamental ideas.

ISBN 0-8218-2800-2

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CONM/316

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