Diagrammatic Morphisms and Applications
Diagrammatic Morphisms and Applications

AMS Special Session on Diagrammatic Morphisms in Algebra, Category Theory, and Topology
October 21–22, 2000
San Francisco State University, San Francisco, California

David E. Radford
Fernando J. O. Souza
David N. Yetter
Editors

American Mathematical Society
Providence, Rhode Island
The 2000 AMS Special Session on "Diagrammatic Morphisms in Algebra, Category Theory, and Topology" was held at San Francisco State University, San Francisco, California, October 21–22, 2000.

2000 Mathematics Subject Classification. Primary 00B25, 16W30, 18Dxx, 57M15, 57M25, 57M27, 83C45; Secondary 15-XX, 16B50, 16S30.

Library of Congress Cataloging-in-Publication Data
AMS Special Session on Diagrammatic Morphisms in Algebra, Category Theory, and Topology (2000 : San Francisco, Calif.)
p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 318)
Includes bibliographical references.
QA169.A483 2000
512.55—dc21 2002033012

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org. Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2003 by the American Mathematical Society. All rights reserved.

The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.
Visit the AMS home page at http://www.ams.org/
10 9 8 7 6 5 4 3 2 1 08 07 06 05 04 03
Contents

Diagrammatic morphisms
  D. N. YETTER  1

Spin foam perturbation theory
  JOHN C. BAEZ  9

Unlinked embedded graphs
  JOHN W. BARRETT  23

Report on cross product bialgebras in braided categories
  YURI BESPALOV AND BERNHARD DRabant  29

Diagrammatic computations for quandles and cocycle knot invariants
  J. SCOTT CARTER, SEIICHI KAMADA, AND MASAHICO SAITO  51

Lax monoids, pseudo-operads, and convolution
  BRIAN DAY AND ROSS STREET  75

Diagrammatic formulation of multi-braided quantum groups
  MICHO ĐURĐEVICH  97

A matrix model for quantum $SL_2$
  CHARLES FROHMAN AND JOANNA KANIA-BARTOSZYNsKA  107

Bi-oriented quantum algebras, and a generalized Alexander polynomial for virtual links
  LOUIS H. KAUFFMAN AND DAVID RADFORD  113

Towards an algebraic characterization of 3-dimensional cobordisms
  THOMAS KERLER  141

Operad of graphs, convolution and quasi Hopf algebra
  ZBIgniew Oziewicz  175

$SU_n$—quantum invariants for periodic links
  JÓZEF H. PRZTYCKI AND ADAM S. SIKORA  199

Weak omega-categories
  ROSS STREET  207
Preface

Diagrammatic (or graphical) morphisms were initially developed as classes of oriented graphs immersed into the plane or 3-space with vertices and arrows labeled by morphisms and objects of categories, respectively. They are often required to satisfy topological and diagrammatic equivalence relations. Higher-dimensional generalizations were introduced later. Diagrammatic morphisms have played a fundamental role in comprehending and visualizing certain categories with structure. They have been of great importance in algebra, low-dimensional topology, and physics. They have also been recognized in classical and quantum information processing and logic. Their use has often shortened statements and proofs. The introduction to this volume, written by David Yetter, is a thorough account of diagrammatic morphisms in a historical perspective.

Some categories of diagrammatic morphisms are structured categories freely generated by a given category. Therefore, diagrammatic morphisms can provide solutions to the coherence problems for some structures. That is to say that one can recognize when two structural morphisms are equal by checking whether or not they correspond to the same diagrammatic morphism. One can also represent diagrammatically a given structured category $C$ as an appropriate quotient of the structured category freely generated by $C$. The articles by Brian Day and Ross Street in this volume study structured categories.

Understanding certain algebraic objects in categories have been greatly helped by the use of diagrammatic morphisms. Universal objects and new algebraic objects have been obtained in this manner. Case in point: Hopf-algebra objects and their generalizations, which are well-explored in this volume.

In quantum topology, functors from categories built out of decompositions of topological spaces to categories with suitable structure have systematically produced invariants, abstracting, and sometimes unifying, various topological constructions. This volume contains some articles in this direction, especially in conjunction with diagrammatic constructions of algebraic objects. As a matter of fact, the simultaneous use of diagrammatic morphisms in algebra and topology has furthered the interplay between these areas. The analysis of what kinds of algebraic and/or categorical structures are suitable ingredients for topological invariants is not rare in the literature.

Diagrammatic methods have been used in physics intensively since the introduction of Feynman diagrams and spin networks. These methods were a strong motivation for the entire subject of diagrammatic morphisms. Applications of diagrammatic techniques to physics are represented here by John Baez’s article.

In spite of the large number of results and constructions obtained via diagrammatic approaches, a good many researchers are not yet familiar with diagrammatic
morphisms, or are not fully appreciative of their usefulness. For this reason, Fer-
nando Souza invited the other two editors to help organize a conference session
dedicated completely to the various aspects of diagrammatic morphisms and their
applications. His intention was to gather people active in that field in order to
bring forth contributions to its consolidation. The result was the *Special Session
on Diagrammatic Morphisms in Algebra, Category Theory, and Topology* at the
2000 Fall Western Section Meeting of the American Mathematical Society (AMS
Meeting # 958) in San Francisco, October 21–22, 2000. Researchers from Australia,
Canada, England, Germany, Mexico, Portugal and, of course, the USA participated
in that special session.

Each article in this volume was refereed by one to three referees.

David E. Radford (University of Illinois at Chicago),
Fernando J. O. Souza (University of Iowa),
David N. Yetter (Kansas State University),
November 2002.
Acknowledgements

David Radford and David Yetter wish to thank the National Science Foundation for support during the time of the special session and most of the preparation of the proceedings volume.

Fernando Souza would like to thank the institutions that employed him during the preparation of this volume and the organization of the corresponding AMS Special Session: the University of Waterloo (Department of Combinatorics and Optimization, Centre for Applied Cryptographic Research, and Institute for Quantum Computation), and the T-6 group of the Los Alamos National Laboratory, respectively. While editing this volume, he was primarily supported by a postdoctoral fellowship sponsored by the projects Applied Cryptography and Quantum Computing (MITACS) and Quantum Computation and Cryptography (PREA) hosted by Michele Mosca.

The editors express gratitude to Sergei Gel’fand and Christine Thivierge of the American Mathematical Society for their patience and care regarding this volume, and Bernard Russo for his diligent handling of the organization of the AMS meeting at which the special session took place.
Program of the Original Session

Session 1 — Saturday A.M.
8:00-8:35 A.M.
958-01-222 Diagrammatic Morphisms, or Directly from Geometry to Algebra and Back Again: An Overview.
David N. Yetter*, dyetter@math.ksu.edu
8:45-9:20 A.M.
958-83-242 n-Categorical Physics.
John C. Baez*, baez@math.ucr.edu
9:30-10:05 A.M.
958-57-94 The Kauffman Bracket Skein as an Algebra of Observables.
Douglas Bullock, bullock@math.idbsu.edu
Charles Frohman, frohman@math.uiowa.edu
Joanna Kania-Bartoszynska*, kania@math.idbsu.edu
10:15-10:50 A.M.
958-57-88 The Yang Mills Measure in the Kauffman Bracket Skein Module.
Doug Bullock, bullock@math.idbsu.edu
Charles D Frohman*, frohman@math.uiowa.edu
Joanna M Kania-Bartoszynska, kania@math.idbsu.edu

Session 2 — Saturday P.M.
3:00-3:35 P.M.
958-57-60 Symplectic Structure on Colorings and Lagrangian Tangles.
Jozef H. Przytycki*, przytyck@gwu.edu
3:45-4:20 P.M.
958-18-183 Knot Invariants, Embedded Graphs and the Category of Representations of the Lorentz Group.
John W. Barrett*, jwb@maths.nott.ac.uk
4:30-5:05 P.M.
958-57-253 Young Tabloids and Representations of Braid Groups.
Xiao-Song Lin*, xl@math.ucr.edu
5:15-5:50 P.M.
958-17-217 Cyclic Operads and Algebra of Chord Diagrams.
Vladimir Hinich, hinich@math.haifa.ac.il
Arkady Vaintrob*, vaintrob@math.uoregon.edu
Session 3 — Sunday A.M.
8:00-8:35 A.M.
958-57-172 Linking of Orientable and Non-orientable Surfaces in 4-dimensions.
J. Scott Carter*, carter@mathstat.usouthal.edu
Seiichi Kamada, kamada@sci.osaka-cu.ac.jp
Masahico Saito, saito@math.usf.edu
Shin Satoh, susato@sci.osaka-cu.ac.jp

8:45-9:20 A.M.
958-57-170 Quandle Cocycle Invariants of Knots and Shadow Colored Arc Diagrams.
J. Scott Carter
Daniel Jelsovsky
Seiichi Kamada
Masahico Saito*, saito@math.usf.edu

9:30-10:05 A.M.
958-53-62 4D TQFT’s and Gerbes.
Marco A. Mackaay*, mmackaay@ualg.pt

Session 4 — Sunday P.M.
3:00-3:35 P.M.
Ross H. Street*, street@math.mq.edu.au

3:45-4:20 P.M.
958-18-142 Unifying Bialgebra Factorizations.
Bernhard K. Drabant*, drabant@addcom.de

4:30-5:05 P.M.
Thomas Kerler*, kerler@math.ohio-state.edu

5:15-5:50 P.M.
9958-16-238 Quantum Clifford Algebras and Multibraided Quantum Groups.
Micho Durdevich*, micho@matem.unam.mx
PHOTOS

(left to right): Berhnard K. Drabant, Arkady Vaintrob, Sergei Tabachnikov (below), Thomas Kerler (above), Charles D. Frohman, Ross H. Street, John W. Barrett.

PHOTOS

John C. Baez, Joanna M. Kania-Bartoszynska, David E. Radford (below), J. Scott Carter (above), Xiao-Song Lin, Marco A. Mackaay, Micho Đurđevićh, Masahico Saito.

Dirk Kreimer.
Titles in This Series

318 David E. Radford, Fernando J. O. Souza, and David N. Yetter, Editors, Diagrammatic morphisms and applications, 2003
316 O. Cornea, G. Lupton, J. Oprea, and D. Tanré, Editors, Lusternik-Schnirelmann category and related topics, 2002
315 Theodore Voronov, Editor, Quantization, Poisson brackets and beyond, 2002
314 A. J. Berrick, Man Chun Leung, and Xingwang Xu, Editors, Topology and Geometry: Commemorating SISTAG, 2002
313 M. Zuhair Nashed and Otmar Scherzer, Editors, Inverse problems, image analysis, and medical imaging, 2002
312 Aaron Bertram, James A. Carlson, and Holger Kley, Editors, Symposium in honor of C. H. Clemens, 2002
311 Clifford J. Earle, William J. Harvey, and Sevín Recillas-Pishmish, Editors, Complex manifolds and hyperbolic geometry, 2002
310 Alejandro Adem, Jack Morava, and Yongbin Ruan, Editors, Orbifolds in mathematics and physics, 2002
309 Martin Guest, Reiko Miyaoka, and Yoshihiro Ohnita, Editors, Integrable systems, topology, and physics, 2002
308 Martin Guest, Reiko Miyaoka, and Yoshihiro Ohnita, Editors, Differentiable geometry and integrable systems, 2002
307 Ricardo Weder, Pavel Exner, and Benoit Grébert, Editors, Mathematical results in quantum mechanics, 2002
306 Xiaobing Feng and Tim P. Schulze, Editors, Recent advances in numerical methods for partial differential equations and applications, 2002
305 Samuel J. Lomonaco, Jr. and Howard E. Brandt, Editors, Quantum computation and information, 2002
304 Jorge Alberto Calvo, Kenneth C. Millett, and Eric J. Rawdon, Editors, Physical knots: Knotting, linking, and folding geometric objects in $\mathbb{R}^3$, 2002
303 William Cherry and Chung-Chun Yang, Editors, Value distribution theory and complex dynamics, 2002
302 Yi Zhang, Editor, Logic and algebra, 2002
301 Jerry Bona, Roy Choudhury, and David Kaup, Editors, The legacy of the inverse scattering transform in applied mathematics, 2002
300 Sergei Vostokov and Yuri Zarhin, Editors, Algebraic number theory and algebraic geometry: Papers dedicated to A. N. Parshin on the occasion of his sixtieth birthday, 2002
299 George Kamberov, Peter Norman, Franz Pedit, and Ulrich Pinkall, Quaternions, spinors, and surfaces, 2002
298 Robert Gilman, Alexei G. Myasnikov, and Vladimir Shpilrain, Editors, Computational and statistical group theory, 2002
297 Stephen Berman, Paul Fendley, Yi-Zhi Huang, Kailash Misra, and Brian Parshall, Editors, Recent developments in infinite-dimensional Lie algebras and conformal field theory, 2002
296 Sean Cleary, Robert Gilman, Alexei G. Myasnikov, and Vladimir Shpilrain, Editors, Combinatorial and geometric group theory, 2002
295 Zhangxin Chen and Richard E. Ewing, Editors, Fluid flow and transport in porous media: Mathematical and numerical treatment, 2002
294 Robert Coquereaux, Ariel García, and Roberto Trinchero, Editors, Quantum symmetries in theoretical physics and mathematics, 2002
TITLES IN THIS SERIES

293 Donald M. Davis, Jack Morava, Goro Nishida, W. Stephen Wilson, and Nobuaki Yagita, Editors, Recent progress in homotopy theory, 2002
292 A. Chenciner, R. Cushman, C. Robinson, and Z. Xia, Editors, Celestial Mechanics, 2002
291 Bruce C. Berndt and Ken Ono, Editors, $q$-series with applications to combinatorics, number theory, and physics, 2001
290 Michel L. Lapidus and Machiel van Frankenhuyzen, Editors, Dynamical, spectral, and arithmetic zeta functions, 2001
289 Salvador Pérez-Esteva and Carlos Villegas-Blas, Editors, Second summer school in analysis and mathematical physics: Topics in analysis: Harmonic, complex, nonlinear and quantization, 2001
287 Marlos A. G. Viana and Donald St. P. Richards, Editors, Algebraic methods in statistics and probability, 2001
284 Gaston M. N’Guérekata and Asamoah Nkwanta, Editors, Council for African American researchers in the mathematical sciences: Volume IV, 2001
283 Paul A. Milewski, Leslie M. Smith, Fabian Waleffe, and Esteban G. Tabak, Editors, Advances in wave interaction and turbulence, 2001
282 Arlan Ramsay and Jean Renault, Editors, Groupoids in analysis, geometry, and physics, 2001
281 Vadim Olshevsky, Editor, Structured matrices in mathematics, computer science, and engineering II, 2001
280 Vadim Olshevsky, Editor, Structured matrices in mathematics, computer science, and engineering I, 2001
278 Eric Todd Quinto, Leon Ehrenpreis, Adel Faridani, Fulton Gonzalez, and Eric Grinberg, Editors, Radon transforms and tomography, 2001
277 Luca Capogna and Loredana Lanzani, Editors, Harmonic analysis and boundary value problems, 2001
276 Emma Previato, Editor, Advances in algebraic geometry motivated by physics, 2001
274 Ken-ichi Maruyama and John W. Rutter, Editors, Groups of homotopy self-equivalences and related topics, 2001
272 Eva Bayer-Fluckiger, David Lewis, and Andrew Ranicki, Editors, Quadratic forms and their applications, 2000
271 J. P. C. Greenlees, Robert R. Bruner, and Nicholas Kuhn, Editors, Homotopy methods in algebraic topology, 2001

For a complete list of titles in this series, visit the AMS Bookstore at www.ams.org/bookstore/.
The technique of diagrammatic morphisms is an important ingredient in comprehending and visualizing certain types of categories with structure. It was widely used in this capacity in many areas of algebra, low-dimensional topology and physics. It was also applied to problems in classical and quantum information processing and logic.

This volume contains articles based on talks at the Special Session, “Diagrammatic Morphisms in Algebra, Category Theory, and Topology”, at the AMS Sectional Meeting in San Francisco. The articles describe recent achievements in several aspects of diagrammatic morphisms and their applications. Some of them contain detailed expositions on various diagrammatic techniques. The introductory article by D. Yetter is a thorough account of the subject in a historical perspective.