Vector Bundles and Representation Theory
Conference on Hilbert Schemes, Vector Bundles and Their Interplay with Representation Theory
April 5–7, 2002
University of Missouri, Columbia

S. Dale Cutkosky
Dan Edidin
Zhenbo Qin
Qi Zhang
Editors
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Preface

The study of Hilbert schemes and vector bundles is a fundamental problem in algebraic geometry. Their connections with physics and representation theory were pioneered in the work of Penrose and Atiyah during the 1970s. They played central roles in the Donaldson theory and the Seiberg-Witten theory from 1983 to 1995. From 1995, physicists working in string theory have speculated many surprising but deep results concerning Hilbert schemes, vector bundles and their interplay with representation theory. For instance, a mathematical version of the $S$-duality conjecture formulated by C. Vafa and E. Witten revealed a beautiful connection between stable vector bundles on algebraic surfaces and representations of certain infinite-dimensional Lie algebras. In the case of rank one, this leads to an elegant relation between the Hilbert schemes of points on algebraic surfaces and the representations of the infinite-dimensional Heisenberg algebras. Another example of recent interplay between vector bundles and representation theory motivated by physics is the relation among principal bundles over elliptic Calabi-Yau manifolds, representation of compact Lie groups and the physics F-theory. These recent physics discoveries have been leading to intensive studies of and rapid advances in the theory of Hilbert schemes, vector bundles and representation theory.

It is under such a background that in April 2002, the Department of Mathematics at the University of Missouri hosted a conference in Columbia, Missouri on Hilbert Schemes, Vector Bundles and Their Interplay with Representation Theory. The meeting brought together both senior and young researchers, including a number of graduate students, in algebraic geometry and representation theory.

Main speakers at the conference were W.-P. Li (HKUST), E. Izadi (University of Georgia), D. Morrison (Duke), K. Oguiso (University of Tokyo), J. Li (Stanford), W. Wang (MSRI & University of Virginia), R. Friedman (Columbia University), and H. Nakajima (Kyoto University). There were nineteen short communications presented by E. Gasparim (New Mexico State University), C.-H. Liu (Harvard), T. Nevins (MSRI), A. Iarrobino (Northeastern), G. J. Pearlstein (UC-Irvine), B. Kotzev (University of Missouri), Y. Kimiko (Kyoto University), D. Arcara (University of Georgia), A. Vitter (Tulane), W. Li (Oklahoma State University), C. Madonna (Univ. Roma 2), A. Caldararu (U. Mass), A. Mavlyutov (Indiana University), T. Luo (UT-Arlington), Y. Kachi (University of Tennessee), E. Markman (U. Mass), B. Purnaprajna (University of Kansas), P. Rao (University of Missouri-St. Louis), and X. Wu (University of South Carolina).

The meeting provided a forum for mathematicians in algebraic geometry and representation theory to meet with colleagues and learn of recent research developments in the focused areas. It also gave graduate students and recent Ph.D's an
opportunity to broaden their mathematical horizons, and promoted the research status of under-represented groups.

This proceedings volume contains 13 papers presented at the conference, and papers inspired by it. All of these papers were carefully refereed, and are in final form. They represent the latest developments in Hilbert schemes, vector bundles and their interplay with representation theory.

The conference was supported by the National Science Foundation under Grant No. 0118343, by the National Security Agency under Grant No. MSPF-02IC-002, and by the Math Department of the University of Missouri-Columbia. We thank them for their generous financial support. We also thank B. Frazier, D. W. Hodge, J. White, and K. Wilson for their help at various stages of the conference, and C. M. Thivierge for her assistance in preparing this proceedings volume.

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This volume contains 13 papers from the conference on “Hilbert Schemes, Vector Bundles and Their Interplay with Representation Theory”. The papers are written by leading mathematicians in algebraic geometry and representation theory and present the latest developments in the field.

Among other contributions, the volume includes several very impressive and elegant theorems in representation theory by R. Friedman and J. W. Morgan, convolution on homology groups of moduli spaces of sheaves on K3 surfaces by H. Nakajima, and computation of the $S^1$ fixed points in Quot-schemes and mirror principle computations for Grassmannians by S.-T. Yau, et al.

The book is of interest to graduate students and researchers in algebraic geometry, representation theory, topology and their applications to high energy physics.