CONTEMPORARY MATHEMATICS

338

Heat Kernels and Analysis on Manifolds, Graphs, and Metric Spaces

Lecture Notes from a Quarter Program on
Heat Kernels, Random Walks, and
Analysis on Manifolds and Graphs
April 16 – July 13, 2002
Emile Borel Centre of the Henri Poincaré Institute
Paris, France

Pascal Auscher
Thierry Coulhon
Alexander Grigor'yan
Editors

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This volume contains elaborated lecture notes of most of the lecture courses taught during the quarter "Heat kernels, random walks and analysis on manifolds and graphs" held at the Emil Borel Centre of the Henri Poincaré Institute, Paris, from April 16 to July 13, 2002.

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Preface

This volume contains elaborated lecture notes of most lecture courses taught during the quarter "Heat kernels, random walks and analysis on manifolds and graphs" held at the Emile Borel Centre of the Henri Poincaré Institute, Paris, from 16 April to 13 July 2002. The scientific program was organized by Pascal Auscher, Gérard Besson, Thierry Coulhon, Alexander Grigor'yan and was sponsored by the Emile Borel Centre, the European network Harmonic Analysis and Related Problems, the University of Cergy-Pontoise, the University of Picardie-Jules Verne, and the Graduate School Economie et Mathématiques Paris-Ouest.

Apart from 17 lecture courses (each 6–12 hours long), the program featured weekly seminar and the following workshops:

(1) April 22–26 "Harmonic analysis and PDEs"
(2) May 28–31 "Heat kernels and analysis on manifolds"
(3) June 26–28 "Analysis on graphs and on metric spaces".

The main purpose of the program was to bring together experts on analysis on different kind of spaces: Euclidean spaces, Riemannian manifolds, Lie groups, infinite dimensional spaces, metric measure spaces, fractals, finitely generated groups, graphs, etc. Recent advances in each of these fields have exhibited certain similarities, and we have felt that intensive communication across the fields is essential for the further development of all of them. At the same time, the quarter gave a unique opportunity for young researchers (postdocs and PhD students) to receive an up-to-date account of the fields directly from the leading experts.

We hope that this volume will be a valuable source both for experts and young researchers by providing them with a wealth of information, which otherwise is scattered in literature or not published at all.

For some reasons, not all lecture courses are presented in this volume. Our colleague and friend Bob Brooks passed away in September 2002 suddenly after a heart attack. He was one of the most active participants in the above quarter and gave a brilliant lecture course on spectral geometry. Bob's notes will be prepared by one of his collaborators for publication in a special volume dedicated to his memory. We shall keep preciously the memory of his presence in Paris.

P. Auscher, T. Coulhon, and A. Grigor'yan
July 2003

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This volume contains the expanded lecture notes of courses taught at the Emile Borel Centre of the Henri Poincaré Institute (Paris) on heat kernels, random walks, and analysis on manifolds and graphs.

In the book, leading experts introduce recent research in their fields. The unifying theme is the study of heat kernels in various situations using related geometric and analytic tools. Topics include analysis of complex-coefficient elliptic operators, diffusions on fractals and on infinite-dimensional groups, heat kernel and isoperimetry on Riemannian manifolds, heat kernels and infinite dimensional analysis, diffusions and Sobolev-type spaces on metric spaces, quasi-regular mappings and p-Laplace operators, heat kernel and spherical inversion on $SL_2(C)$, random walks and spectral geometry on crystal lattices, isoperimetric and isocapacitary inequalities, and generating function techniques for random walks on graphs.

This volume is suitable for graduate students and research mathematicians interested in random processes and analysis on manifolds.

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