Wavelets, Frames and Operator Theory

Focused Research Group Workshop on Wavelets, Frames and Operator Theory

January 15–21, 2003
University of Maryland
College Park, Maryland

Christopher Heil
Palle E.T. Jorgensen
David R. Larson
Editors
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Preface

This book grew out of the Special Session on Wavelets, Frames, and Operator Theory that we organized at the 2003 Annual Meeting of the AMS in Baltimore, January 15–18, 2003, and an immediately following NSF-sponsored workshop organized by John Benedetto at The University of Maryland, January 19–21, 2003. Both events were associated with the NSF Focused Research Group (FRG) of which we are a part, and whose other members are Akram Aldroubi, Lawrence W. Baggett, John J. Benedetto, Gestur Ólafsson, and Yang Wang. The speakers in the Special Session and the Maryland workshop were invited to contribute papers, and this volume is the very pleasant result.

We hope that those events and more like them that have since taken place or are planned for the future, and the present book itself, will act as a catalyst, encouraging members of our community to work on one or more of the many facets of the intertwined subjects of wavelets, frames, and operator theory. Some of the papers included here focus more on one of the three areas than the other two, but all hint at exciting connections and interrelationships. They stand at the crossroads of harmonic analysis, operator theory, and applied mathematics. Some papers are theoretical, some applied, but most are a mix of theory and applications, each inspiring the other. Wavelets and frames have emerged as significant tools in mathematics and in technology over the past two decades. They interact with harmonic analysis, with operator theory, and with a host of applications. In their primitive form, both wavelets and frames originate with the vector space notion of a basis. They are used in the analysis of functions, and the functions make up infinite-dimensional spaces, typically Hilbert spaces. While many wavelet constructions yield orthonormal bases, frames by their very nature, including many interesting classes of wavelets, satisfy conditions which are more general than the familiar orthogonality relations. Historically, operator theory has played a big part in the analysis of both wavelets and frames, and we hope to highlight this feature in our collection of papers.

The workshops, the research, and the publication of this volume were supported in part by our FRG grant from the National Science Foundation.\(^1\) It is also a pleasure to thank Brian Treadway, whose assistance was essential to the smooth

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\(^1\)DMS-0139759 Collaborative Research: Focused Research on Wavelets, Frames, and Operator Theory. Description: In this project, fundamental problems are addressed in wavelet theory, non-uniform sampling, frames, and the theory of spectral-tile duality. These problems are inextricably interwoven by concept and technique. Operator theory provides the major unifying framework, combined with an integration of ideas from a diverse spectrum of mathematics including classical Fourier analysis, noncommutative harmonic analysis, representation theory, operator algebras, approximation theory, and signal processing. For example, the construction, implementation, and ensuing theory of single dyadic orthonormal wavelets in Euclidean space requires significant input from all of these disciplines as well as deep spectral-tile results.
completion of this volume. Brian managed the correspondence with referees and authors, organized the many drafts of papers, helped bring them into the \TeX format for the book series, and managed and assisted us in numerous typesetting issues.

Christopher Heil, Palle E. T. Jorgensen, and David R. Larson
September 21, 2003
Speakers at the two sessions of the 2003 Annual Meeting of the American Mathematical Society, held in Baltimore, and the Wavelet Workshop held in College Park, Maryland, out of which the papers in this volume arose.

Symbols are used in the list to designate the specific sessions, as follows:


(C): AMS Session on “Operator Theory”, at the same Annual Meeting;

(W): NSF Focused Research Group Workshop on the theme “Modulation Spaces and the Continuous Wavelet Transform” (University of Maryland, College Park, MD, January 19–21, 2003).

The ten speakers marked (C) had requested to be part of our Special Session, but unfortunately could not be included due to lack of space. The AMS incorporated these speakers into a related session of contributed talks. We thank the AMS for kindly honoring our request to include them in this way, and we consider them informally included in our session for the purpose of this volume.

Affiliations follow the session symbols for each speaker. Where more than one affiliation is listed, the first is from the time of the sessions, and the others are later locations.

Akram Aldroubi (S), Vanderbilt University
http://www.math.vanderbilt.edu/~aldroubi/
“Non uniform sampling and reconstruction in irregular spaces”

Radu Balan (S), Siemens Corporate Research
“Measure function and redundancy of Weyl-Heisenberg multiframes and superframes”

Robert Benedetto (S), Amherst College
http://www.cs.amherst.edu/~rlb/
“Wavelets on \( p \)-adic fields and related groups”

Ola Bratteli (S), University of Oslo
http://www.math.uio.no/~bratteli/
“Global structure of the scaling-wavelet variety”

Peter G. Casazza (S), University of Missouri, Columbia
http://www.math.missouri.edu/~pete/
“Existence and construction of finite frames with a given frame operator”
LIST OF SPEAKERS

Ingrid Daubechies (S), Princeton University
http://www.princeton.edu/-icd/
"An iterative algorithm for ill-posed inverse problems where the object has a sparse wavelet expansion"

Dorin Dutkay (S), University of Iowa
http://www.math.uiowa.edu/-ddutkay/
"The local trace function of shift invariant subspaces"

Hans G. Feichtinger (S, W—plenary talk), University of Vienna
http://www.univie.ac.at/NuHAG/FEI/
"Approximation of linear operators by Gabor multipliers" (S)

Matthew C. Fickus (S, W), Cornell University
http://www.math.cornell.edu/People/Postdocs/fickus.html
"Frames in communications" (S)
"A physical interpretation for finite tight frames" (W)

Yevgeniy V. Galperin (C), Sacred Heart University
"Embeddings of Fourier-Lebesgue spaces into modulation spaces: Optimality of sufficient conditions"

Joel K. Glenn (C), Colorado College
"Frequency estimation and vortex analysis using wavelet coefficients"

Karlheinz Gröchenig (S, W—plenary talk), University of Connecticut
http://www.math.uconn.edu/-groch/
"Localization of frames" (S, W)

Christopher Hammond (C), University of Virginia
"On the norm of a composition operator with linear fractional symbol"

Deguang Han (S), University of Central Florida
http://pegasus.cc.ucf.edu/-dhan/main.html
"Operator parametrization and tight frame approximations"

Doug Hardin (S), Vanderbilt University
http://math.vanderbilt.edu/-hardin/
"Continuous orthogonal wavelets on semi-regular triangulations"

Denise Jacobs (C), United States Military Academy
http://www.dean.usma.edu/math/people/jacobs/
"Orthogonal wavelets in higher dimensions"

Brody Johnson (W), Georgia Institute of Technology; Saint Louis University
http://euler.slu.edu/Dept/Faculty/johnson/
"Oversampling wavelet frames"

Norbert Kaiblinger (S), University of Vienna
http://www.mat.univie.ac.at/-kaib
"Varying the lattice of Gabor frames"

Keri Kornelson (S), Texas A & M University
http://www.math.tamu.edu/-keri.kornelson/
"Ellipsoidal tight frames"

Gitta Kutyniok (S, W), University of Paderborn
http://www-math.uni-paderborn.de/-gittak/
"Density of weighted wavelet frames" (S)
"A qualitative uncertainty principle for functions generating a Gabor frame on LCA groups" (W)
LIST OF SPEAKERS

Demetrio Labate (S, W), Washington Univ., St. Louis; North Carolina State Univ.
http://www4.ncsu.edu:8030/~dlabate/
“A unified theory of reproducing function systems” (S)
“Oversampling of affine systems” (W)

Jeffrey C. Lagarias (S), AT&T Labs—Research
http://www.research.att.com/~jcl/
“A family of piecewise-linear plane maps and associated nonlinear difference operators of Schrödinger type”

Mark Lammers (S, W), Western Washington U.; U. of North Carolina, Wilmington
http://people.uncw.edu/lammersm/
“Wilson bases and convolution” (S)

Zeph Landau (S, W), Mathematical Sciences Research Institute; Microsoft Corp.
“Densities of frames” (S)
“Measuring sequences, subspaces, and frames” (W)

Ursula Molter (S), Universidad de Buenos Aires
http://mate.dm.uba.ar/~umolter/
“Optimal shift-invariant spaces”

Krzysztof Nowak (C), Drexel University
“Best projections of Gabor multiplier type”

Kasso Okoudjou (S, W), Georgia Institute of Technology; Cornell University
http://www.math.cornell.edu/~kasso/
“Bilinear pseudodifferential operators on modulation spaces” (S)
“Gabor analysis in amalgam spaces” (W)

Gestur Ólafsson (S), Louisiana State University
http://www.math.lsu.edu/~olafsson/
“Frames and groups”

Judith Packer (S), University of Colorado, Boulder
http://spot.colorado.edu/~packer/
“An analogue of the Bratteli-Jorgensen loop group action for m-systems in the GMRA setting”

Manos Papadakis (S), University of Houston
http://www.math.uh.edu/~mpapadak/
“Symmetric univariate QM filters with Gaussian decay”

Alexander Powell (S), University of Maryland; Princeton University
“A (p, q) weighted version of a theorem of J. Bourgain”

T. Gabriel Prajitura (C), State University of New York, Brockport
“Approximation by countably hypercyclic operators”

Dmitry Ryabogin (C), University of Missouri, Columbia
http://www.math.missouri.edu/~ryabs/
“The Calderón reproducing formula and rough singular integrals”

Ziemowit Rzeszotnik (S), University of Texas, Austin
http://www.ma.utexas.edu/users/zioma/
“Unitary operators preserving wavelets”

Songkiat Sumetkijakan (W), University of Maryland; Chulalongkorn University
http://pioneer.netser.v.chula.ac.th/~ssongkia/
“On the neighborhood-mapping construction of wavelet sets in $\mathbb{R}^d$”
Qiyu Sun (S), University of Houston; University of Central Florida
http://gauss.math.ucf.edu/~qsun/
“Symmetric univariate QM filters with Gaussian decay II”

David Walnut (S), George Mason University
“Local reconstruction from averages”

Ying Wang (C, W), Marywood University
“On joint perturbations of Gabor frames” (C)
“On perturbations of irregular Gabor frames” (W)

Eric Weber (S, W), University of Wyoming; Iowa State University
http://www.math.iastate.edu/ esw/
“Superwavelets and generalized multiresolution analysis” (S)
“Orthogonal frames of translates” (W)

Guido Weiss (S, W—plenary talk), Washington University, St. Louis
http://www.math.wustl.edu/~guido/
“On the connectivity of wavelets” (S)
“A unified theory for the characterization of reproducing systems” (W)

Janine Wittwer (C), Williams College
http://www.williams.edu/Mathematics/jwittwer/
“Wavelets and Bellman functions”

Richard A. Zalik (S), Auburn University
http://www.auburn.edu/~zalikri/zalikri.html
“On MRA Riesz wavelets”

Shijun Zheng (C, W), University of Maryland; Louisiana State University
“Littlewood-Paley theory associated with Schrödinger operators with hyperbolic secant potentials” (C)
“Schrödinger operator, Besov spaces, and wavelet computations for thin film image processing” (W)

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This book grew out of a special session on Wavelets, Frames and Operator Theory held at the Joint Mathematics Meetings in Baltimore and a National Science Foundation-sponsored workshop held at the University of Maryland. Both events were associated with the NSF Focused Research Group.

In the past two decades, wavelets and frames have emerged as significant tools in mathematics and technology. They interact with harmonic analysis, operator theory, and a host of other applications. This volume includes both theoretical and applied papers highlighting the many facets of these interconnected topics. It is suitable for graduate students and researchers interested in wavelets and their applications.