

CONTEMPORARY MATHEMATICS

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Spectral Geometry of Manifolds with Boundary and Decomposition of Manifolds

Proceedings of the Workshop on
Spectral Geometry of Manifolds with
Boundary and Decomposition of Manifolds
Roskilde University, Roskilde, Denmark
August 6–9, 2003

Bernhelm Booß-Bavnbek
Gerd Grubb
Krzysztof P. Wojciechowski
Editors



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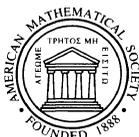
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Preface

On August 6-9, 2003, a workshop was held on *Spectral Geometry of Manifolds with Boundary and Decomposition of Manifolds* at Roskilde University in Denmark.

We envisioned a small meeting with a limited number of lectures and plenty of space and time for interaction between mathematicians having different perspectives on the topic. It is our feeling that the meeting was successful in achieving its primary goals. We think that the spirit of the meeting, and the extremely positive and creative atmosphere among the participants, are reflected in these Proceedings.

The papers in this volume are devoted to presenting developments in the subjects from an interlaced geometric and analytical point of view. They cover a wide variety of topics, from recent advances in index theory and more general theory of spectral invariants on closed manifolds and manifolds with boundary, to applications of those invariants in geometry, topology, and physics.

The papers are grouped in four parts. Part I gives an overview of the subject from various points of view. The survey of Vassilevich places the study of spectral invariants in its physics context in Quantum Field Theory, and Esposito explains recent results on local (differential) problems in Quantum Gravity. Grubb reviews the current results and analysis methods for problems with nonlocal (pseudodifferential projection) boundary conditions.

Part II deals with spectral invariants such as traces, indices and determinants. The paper by Grubb gives a novel deduction of Kontsevich and Vishik's canonical trace on closed manifolds by resolvent methods, obtaining additional features for symbols with a parity or a log-polyhomogeneity property. Lee provides an asymptotic expansion of the zeta-determinant, in terms of the cylinder length, of a Laplacian on a manifold stretched by a cylinder, with various boundary conditions. Park and Wojciechowski show how the quotient between the zeta-determinants of Neumann and Dirichlet problems for a Laplacian identifies with the zeta-determinant of the Dirichlet-to-Neumann operator living on the boundary; this is done precisely and not only in the adiabatic limit.

Part III is concerned with general geometric and topological questions. Boden, Herald and Kirk focus on the spectral flow calculations that constitute an important technical issue in the construction of gauge theoretic Casson-like invariants on 3-manifolds. Leichtnam and Piazza work out a generalization of the classical cut-and-paste result for the signature of closed manifolds to the case of higher signatures of foliated bundles. Lesch settles a variety of topological questions regarding the space of (generally unbounded) self-adjoint Fredholm operators in complex Hilbert space, and shows that there are no other additive and normalized homotopy invariants of paths of such operators than spectral flow. On the basis of their previous work

on equivariant Seiberg–Witten Floer cohomologies, Marcolli and Wang deduce a collection of topological invariants for rational homology 3-spheres.

Finally, Part IV deals specifically with problems on manifolds with singularities. Loya shows how Atiyah–Patodi–Singer problems on manifolds with boundary are turned into special cases of problems in the b -calculus (for singular manifolds), for which powerful tools are brought into play in the study of index and eta invariants. Nazaikinskii, Rozenblum, Savin and Sternin attack index problems on manifolds with singularities by methods from K -theory of algebras and cyclic cohomology. The manifolds discussed by Nistor are singular in the sense of having a polyhedral structure at infinity; here spectra and conditions for Fredholmness are determined.

We thank the authors for their contributions, the PDE Network of The Danish Science Research Council for financial support, Roskilde University for their hospitality, and the American Mathematical Society for encouragement and help in preparing this volume.

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In recent years, increasingly complex methods have been brought into play in the treatment of geometric and topological problems for partial differential operators on manifolds. This collection of papers, resulting from a Workshop on Spectral Geometry of Manifolds with Boundary and Decomposition of Manifolds, provides a broad picture of these methods with new results.

Subjects in the book range from spectral flow calculations and cut-and-paste considerations, via pseudodifferential methods, asymptotic expansions and variational arguments, to singular manifold theories and K -theoretic cohomological strategies. They lead to results on determinants, heat kernels, general trace, index and higher signature formulas, low-dimensional topological invariants, as well as on the structure of the manifolds and operators involved. Moreover, the approaches and results are placed in a physics context by two reviews on the applications in quantum field theory, respectively quantum gravity.

This book is suitable for graduate students and researchers interested in spectral problems in geometry.

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