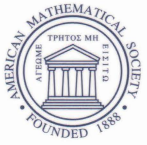
CONTEMPORARY MATHEMATICS

367

Geometric Evolution Equations

National Center for Theoretical Sciences Workshop
on Geometric Evolution Equations
National Tsing Hua University, Hsinchu, Taiwan
July 15–August 14, 2002

Shu-Cheng Chang
Bennett Chow
Sun-Chin Chu
Chang-Shou Lin
Editors



American Mathematical Society

http://dx.doi.org/10.1090/conm/367

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Proceedings of the National Center for Theoretical Sciences Workshop on Geometric Evolution Equations held at National Tsing Hua University, Hsinchu, Taiwan, July 15–August 14, 2002.

2000 *Mathematics Subject Classification.* Primary 53C44, 53C21, 35K55, 57M50, 35K57, 53C42, 53C43, 58J05, 53C35, 58J35.

Library of Congress Cataloging-in-Publication Data

National Center for Theoretical Sciences Workshop on Geometric Evolution Equations (1st : 2002 : Hsin-chu shih, Taiwan)

Geometric evolution equations : National Center for Theoretical Sciences Workshop on Geometric Evolution Equations, National Tsing Hua University, Hsinchu, Taiwan, July 15–August 14, 2002 / Shu-Cheng Chang . . . [et al.], editors.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 367)

Includes bibliographical references.

ISBN 0-8218-3361-8 (alk. paper)

1. Evolution equations, Nonlinear—Numerical solutions—Congresses. 2. Geometry, Algebraic—Congresses. I. Chang, Shu-Cheng, 1959– II. Title. III. Contemporary mathematics (American Mathematical Society) ; v. 367.

QA377.N35 2004

515′.353—dc22 2004046440

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10 9 8 7 6 5 4 3 2 1 10 09 08 07 06 05

Contents

Preface vii

Photo of Conference Participants viii

Program of the Conference ix

Singularities at $t = \infty$ in equivariant harmonic map flow
SIGURD ANGENENT AND JOOST HULSHOF 1

Recent developments on the Calabi flow
SHU-CHENG CHANG 17

Stability of the Kähler-Ricci flow at complete non-compact Kähler Einstein metrics
ALBERT CHAU 43

A survey of Hamilton's program for the Ricci flow on 3-manifolds
BENNETT CHOW 63

Basic properties of gradient Ricci solitons
SUN-CHIN CHU 79

Numerical studies of the behavior of Ricci flow
DAVID GARFINKLE AND JAMES ISENBERG 103

Convex solutions of fully nonlinear elliptic equations in classical differential geometry
PENGFEI GUAN AND XINAN MA 115

Density estimates for minimal surfaces and surfaces flowing by mean curvature
ROBERT GULLIVER 129

An introduction to the Ricci flow neckpinch
DAN KNOPF 141

Monotonicity and Kähler-Ricci flow
LEI NI 149

Deforming Lipschitz metrics into smooth metrics while keeping their curvature operator non-negative
MILES SIMON 167

Liouville properties on Kähler manifolds
LUEN-FAI TAM 181

Expanding embedded plane curves
DONG-HO TSAI 189

Remarks on a class of solutions to the minimal surface system
MU-TAO WANG 229

Preface

The first Workshop on Geometric Evolution Equations was held in the National Center for Theoretical Sciences at National Tsing Hua University in Hsinchu, Taiwan from July 15 to August 14, 2002. It was a fun and exciting conference and we hope that the papers in this volume will convey some of the spirit of the conference. There are 14 papers contained in this volume on various topics in geometric evolution equations and related fields. These topics range from the minimal surface equation, mean curvature flow, harmonic map flow, Calabi flow, Ricci flow (including a numerical study), Kähler-Ricci flow, function theory on Kähler manifolds, flows of plane curves, convexity estimates, and the Christoffel-Minkowski problem.

The organizers would like to express their gratitude to NCTS staff members Wendy Huang, Vickey Sun, and Chia-Yu Twu for all of their help related to the workshop. We are indebted to Professor Dong-Ho Tsai for his help in making local arrangements. We also would like to thank Professors Jing Yu and Shing-Tung Yau for their support of the program. Thanks to Edward Dunne, Jennifer Sharp, and Christine Thivierge at the AMS for all of their assistance and encouragement in making the publication of this volume possible. We are grateful to the NCTS and Acer Foundation for their financial support of the workshop.

B.C. would like to thank Classic Dimension for encouragement and support.

Shu-Cheng Chang
Bennett Chow
Sun-Chin Chu
Chang-Shou Lin

2002 NCTS Workshop on Geometric Evolution Equations
July 15-August 14, 2002
Place: Lecture Room B of NCTS, 4F
Sponsor & Support : National Center for Theoretical Sciences
Department of Mathematics, National Tsing-Hua University

Program of the Conference

Week 1

Tuesday (July 16, 2002)

10:00 JIM ISENBERG: On the absence of restrictions on the spatial topology of globally hyperbolic solutions of Einstein's equations

1:00 ALBERT CHAU: Convergence of the Kähler-Ricci flow on non-compact manifolds

Wednesday (July 17, 2002)

10:00 LUEN-FAI TAM: Gap theorems on Kähler manifolds

1:00 BEN CHOW: Survey of Ricci flow

Thursday (July 18, 2002)

10:00 DAN KNOPF: An injectivity radius estimate for sequences of solutions to the Ricci flow having almost nonnegative curvature operators

1:00 HUAI-DONG CAO: On translating Kähler-Ricci solitons and dimension reduction

Week 2

Tuesday (July 23, 2002)

10:00 ROBERT GULLIVER: A density estimate for minimal surfaces

1:00 LEI NI: Monotonicity and Kähler-Ricci flow on complete manifolds

Wednesday (July 24, 2002)

10:00 MU-TAO WANG: Gradient estimates for mean curvature flows

1:00 Informal problem session on Kähler-Ricci flow

Thursday (July 25, 2002)

10:00 MILES SIMON: Ricci flow of C^0 metrics

1:00 JIAPING WANG: Stable minimal hypersurfaces in nonnegatively curved space

3:00 XI-NAN MA: Convexity in Hessian and curvature equations

Friday (July 26, 2002)

10:00 PENG LU: On the asymptotic scalar curvature ratio of complete Type I-like ancient solutions to the Ricci flow on noncompact 3-manifolds

Week 3

Tuesday (July 30, 2002)

10:00 SIGURD ANGENENT: Some open problems in curve shortening and related PDEs

1:00 TOM ILMANEN: An example of flowing through singularities in Kähler-Ricci flow

Wednesday (July 31, 2002)

10:00 Mao-Pei Tsui: Long time existence and convergence for mean curvature flow for graphs

1:00 Informal problem session on mean curvature flow

Thursday (August 1, 2002)

10:00 Shu-Cheng Chang: The Calabi flow on 4-manifolds

1:00 Informal problem session on Riemannian Ricci flow

Week 4

Tuesday (August 6, 2002)

10:00 DONG-HO TSAI: Expanding immersed convex plane curves

1:00 TOM ILMANEN: Informal problem session on the mean curvature flow

Wednesday (August 7, 2002)

10:00 BING CHENG: Introduction to LYH-type Harnack estimates

1:00 DAVID GLICKENSTEIN: A (pre) compactness property of solutions to the Ricci flow in the absence of injectivity radius estimates

Titles in This Series

367 **Shu-Cheng Chang, Bennett Chow, Sun-Chin Çhu, and Chang-Shou Lin, Editors,** Geometric evolution equations, 2005

366 **Bernhelm Booß-Bavnbek, Gerd Grubb, and Krzysztof P. Wojciechowski, Editors,** Spectral geometry of manifolds with boundary and decompositon of manifolds, 2005

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The Workshop on Geometric Evolution Equations was a gathering of experts that produced this comprehensive collection of articles. Many of the papers relate to the Ricci flow and Hamilton's program for understanding the geometry and topology of 3-manifolds.

The use of evolution equations in geometry can lead to remarkable results. Of particular interest is the potential solution of Thurston's Geometrization Conjecture and the Poincaré Conjecture. Yet applying the method poses serious technical problems. Contributors to this volume explain some of these issues and demonstrate a noteworthy deftness in the handling of technical areas.

Various topics in geometric evolution equations and related fields are presented. Among other topics covered are minimal surface equations, mean curvature flow, harmonic map flow, Calabi flow, Ricci flow (including a numerical study), Kähler-Ricci flow, function theory on Kähler manifolds, flows of plane curves, convexity estimates, and the Christoffel-Minkowski problem.

The material is suitable for graduate students and researchers interested in geometric analysis and connections to topology.

Related titles of interest include *The Ricci Flow: An Introduction*.

ISBN 0-8218-3361-8

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CONM/367

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