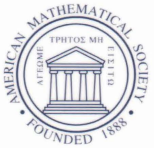
CONTEMPORARY MATHEMATICS

370

The p-Harmonic Equation and Recent Advances in Analysis

IIIrd Prairie Analysis Seminar
October 17–18, 2003
Kansas State University
Manhattan, Kansas

Pietro Poggi-Corradini
Editor



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The spaces we need most are spaces we haven't discovered yet.

Tadeusz Iwaniec, October 2003.

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Preface

This volume comprises all the contributed papers for the IIIrd Prairie Analysis Seminar, which took place on October 17-18, 2003, at Kansas State University in Manhattan, Kansas. The Prairie Analysis Seminar is a yearly event organized jointly by Kansas State University and the University of Kansas. Visit the web-site *http://www.math.ksu.edu/main/events/special/onetime/pas3* for more information on this event.

One of the unique features of this seminar is that the organizers invite one main speaker who is then asked to invite two more speakers of his/her choice. This allows the main speaker to give the conference a strong personal imprint. The organizers then issue an open call for contributed talks, which attracts several other speakers.

At the IIIrd Prairie Analysis Seminar the Main Speaker was Tadeusz Iwaniec, "John Raymond French" Distinguished Professor of Mathematics at Syracuse University, author of a recent important book with G. Martin: *Geometric Function Theory and Nonlinear Analysis*, and winner of the Prix 2001 Institut Henri-Poincaré Gauthier-Villars for the paper "Quasiharmonic Fields", *Ann. Inst. H. Poincaré Analyse Nonlineaire* (2001) 519-572. Famous for his long list of coauthors, Professor Iwaniec's interests range over

- Linear and Nonlinear Elliptic PDEs (Very weak solutions, L^p- regularity theory, Orlicz-Sobolev type estimates, Differential forms).
- The Calculus of Variations (Weak convergence methods, Jacobian determinants and null Lagrangians, Polyconvex and quasiconvex functionals).
- Deformations of Finite Distortion (Functions of one complex variable, Quasiregular mappings in $\mathbb{R}^n$ and the governing PDEs, Topological properties of weakly differentiable mappings between Riemannian manifolds).
- Harmonic Analysis Methods (Singular integrals –sharp estimates–, Interpolation and nonlinear commutators, Hardy spaces).
- Applications in applied mathematics (nonlinear elasticity, material science, microstructure of crystals, and so forth).

The two other invited speakers chosen by T. Iwaniec were John Lewis from the University of Kentucky and Juan Manfredi from the University of Pittsburgh.

J. Lewis's interests range from classical complex analysis to the theory of quasiregular maps, potential theory, elliptic and parabolic equations, Navier-Stokes equations, and many other topics. Most notable is his recent contribution to the solution of Kato's conjectures for elliptic differential operators.

J. Manfredi's work is on the partial differential equations that govern the nonlinear potential theory of quasi-regular mappings in higher dimensions and their subelliptic extensions. His more recent work has focused on the infinity-Laplacian equation.

Hence the field of analysis represented at the Third Prairie Analysis Seminar is an ever-changing discipline with deep roots in classical one-variable complex analysis and partial differential equations. Its more recent aspects touch on very advanced tools from functional analysis, potential theory, and calculus of variations. We hope that the articles that follow will give an idea of the many directions that are being explored.

Sincere thanks must be extended to the founders of the Prairie Analysis Seminar: Marianne Korten and Charles Moore of Kansas State University, and Estela Gavosto and Rodolfo Torres of the University of Kansas, who also helped with the organizational details and in obtaining the necessary funding. The conference was generously sponsored by the Departments of Mathematics at Kansas State University and at the University of Kansas; by the Mathematical Sciences Research Institute in Berkeley, California; and by the National Science Foundation, Grant DMS-0349394. Finally, we are deeply grateful for the help of our secretarial staff, especially that of Sheree Walsh.

Pietro Poggi-Corradini
Manhattan, KS, June 2004

Full Program of the IIIrd Prairie Analysis Seminar

Friday October 17

12:00-12:25 Registration

12:25-12:30 Welcome remarks

12:30-12:50 Artem Zvavitch, University of Missouri-Columbia
The Busemann-Petty problem for Gaussian measures

12:55-1:15 Roger W. Barnard, Texas Tech University
Minimal harmonic measure on complimentary regions

1:20-1:40 Brock Williams, Texas Tech University
Constructing Conformal Maps of Physical Surfaces Using Circle Packings

1:45-2:05 Sergei Merenkov, University of Michigan
Determining biholomorphic equivalence of manifolds from their semigroups of holomorphic self-maps

2:20-3:10 Tadeusz Iwaniec, Syracuse University
p-harmonic equations I

3:10-3:30 Tea time

3:30-4:20 John Lewis, University of Kentucky
Symmetry Theorems and Uniform Rectifiability

4:30-4:50 Peter Hasto, University of Michigan
Variable exponent Lebesgue and Sobolev spaces

4:55-5:15 Luigi D'Onofrio, Universita di Napoli and Syracuse University
The p-Harmonic Transform Beyond its Natural Domain of Definition, Interpolation and Continuity

5:20-5:40 James Peirce, UC Davis
Results on the Lagrangian averaged Navier-Stokes Equations

5:45-6:05 Caroline Sweezy, New Mexico State University
Subspaces of weak L-infinity

6:10-6:30 Mikil Foss, Kansas State University
A condition sufficient for partial regularity of minimizers in two-dimensional nonlinear elasticity

8:00 - ... After-dinner party at Emmily's and Pietro's home

Saturday October 18

8:00-8:20 Leonid Kovalev, Washington University in St. Louis
Comparison theorems for the one-dimensional Schrödinger equation

8:25-8:45 Virginia Naibo, University of Kansas
The universal maximal operator on special classes of functions

8:50-9:10 Genevra Neumann, Kansas State University
Valence of Harmonic Functions

9:15-9:35 Chiara Frosini, Universita di Firenze
Holomorphic Dynamics on bounded domains

9:35-9:50 Coffee break

9:50-10:15 Gerard Ornas, McNeese State University
Extremal Values for a Class of Functionals over Hyperbolically Convex Functions

10:20-10:40 Robert Smits, New Mexico State University
Heat Kernels in Some Self-Similar Domains

10:45-11:05 Christian Wolf, Wichita State University
Measures of maximal dimension for hyperbolic diffeomorphisms

11:10-11:30 Ivan Blank, University of Louisville
Eliminating Mixed Asymptotics in Obstacle Type Free Boundary Problems

11:30 - 12:30 Lunch provided by the Department of Mathematics

12:30-12:50 Diego Maldonado, University of Kansas
Properties of the solutions to the Monge-Ampere equation

12:55-1:15 Byung-Geun Oh, Purdue University
Zeros of the Derivatives of Faber Polynomials Associated with a Universal Covering Map

1:20-1:40 John Ryan, University of Arkansas
Dirac operators, automorphic forms and Hardy spaces on some conformally flat manifolds

1:45-2:05 Jani Onninen, University of Michigan
Mappings of finite distortion: The sharp modulus of continuity

2:20-3:10 Tadeusz Iwaniec, Syracuse University
p-harmonic equations II

3:10-3:30 Tea time

3:30-4:20 Juan Manfredi, University of Pittsburgh
p-Harmonic functions in Euclidean space and in the Heisenberg group

4:30-4:50 Thomas Bieske, University of South Florida
Absolute Minimizers and Infinite Harmonic Functions in Carnot Groups

4:55-5:15 Petronela Radu, Carnegie Mellon University
Weak solutions of semilinear wave equations

5:20-5:40 Alexander Stokolos, DePaul University Chicago
A note on the Gurov-Reshetnyak Lemma

5:45-6:05 Malgorzata Stawiska, Purdue University
Riemann- Hurwitz formula and Morse theory

6:10-6:30 Clint Richardson, Stephen F. Austin State University
Concentration of Area in Half-planes

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Comprised of papers from the IIIrd Prairie Analysis Seminar held at Kansas State University, this book reflects the many directions of current research in harmonic analysis and partial differential equations. Included is the work of the distinguished main speaker, Tadeusz Iwaniec, his invited guests John Lewis and Juan Manfredi, and many other leading researchers.

The main topic is the so-called p-harmonic equation, which is a family of nonlinear partial differential equations generalizing the usual Laplace equation. This study of p-harmonic equations touches upon many areas of analysis with deep relations to functional analysis, potential theory, and calculus of variations.

The material is suitable for graduate students and research mathematicians interested in harmonic analysis and partial differential equations.

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