CONTEMPORARY MATHEMATICS

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Groups, Languages, Algorithms

AMS-ASL Joint Special Session on
Interactions between Logic, Group Theory,
and Computer Science
January 16–19, 2003
Baltimore, Maryland

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Editor

American Mathematical Society

http://dx.doi.org/10.1090/conm/378

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2000 *Mathematics Subject Classification.* Primary 20B40, 20E05, 20F28, 81P68; Secondary 68Q05, 68Q17, 68Q42, 68Q45, 68T05.

Library of Congress Cataloging-in-Publication Data

AMS-ASL Joint Special Session on Interactions between Logic, Group Theory, and Computer Science (2003 : Baltimore, MD)

Groups, Languages, Algorithms : AMS-ASL Joint Special Session on Interactions between Logic, Group Theory, and Computer Science, January 16–19, 2003, Baltimore, Maryland /Alexandre V. Borovik, editor.

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; 378)

Includes bibliographical references.

ISBN 0-8218-3618-8 (alk. paper)

1. Group theory—Congresses. 2. Finite groups—Congresses. 3. Infinite groups—Congresses. I. Borovik, Alexandre. II. Title. III. Contemporary mathematics (American Mathematical Society) ; v. 378.

QA174.A64 2003
512′.2—dc22 2005043610

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10 9 8 7 6 5 4 3 2 1   10 09 08 07 06 05

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Preface

This volume is loosely based around the major themes of the AMS/ASL Joint Special Session on "Interactions Between Logic, Group Theory and Computer Science" held in Baltimore, Maryland, in January 2003. I wish to express my thanks to the American Mathematical Society and Association for Symbolic Logic for their invitation to organize the session and for their support, which allowed this unusual interdisciplinary meeting to take place.

Since the pioneering works of Novikov and Maltsev, group theory was a testing ground for mathematical logic in its many manifestations, from the theory of algorithms to model theory. This interaction between logic and group theory led to many prominent results which enriched both disciplines. In this volume, we collect under one cover several papers devoted to the development of technique for the group theory/logic interface. They complement the previous volume, "Computational and Experimental Group Theory" (vol. 349 of *Contemporary Mathematics*), which also arose from the Baltimore Meeting but concentrated more on a similar interaction between group theory and computer science.

The first paper in the volume, by Robert Gilman, is a detailed survey of the state of art in the theory of formal languages as applied to groups. Formal languages originated as models of spoken and written languages. Subsequently they proved useful in analyzing programming languages, and more recently connections with group theory have begun to emerge. The survey concentrates on the simplest classes of languages, namely regular, context free and indexed languages; some other classes are mentioned briefly. No knowledge of formal languages is assumed on the part of the reader. The exposition emphasizes the algebraic aspects of the subject at the expense of those related to programming; in particular, the language classes are defined in terms of monoids, one for each class.

The next paper, by Myasnikov, Remeslennikov and Serbin, dramatically expands the language metaphor: here, the elements of Lyndon's free $\mathbb{Z}[t]$-group $F^{\mathbb{Z}[t]}$ are represented by *infinite* words with a regular free Lyndon length function on $F^{\mathbb{Z}[t]}$ with values in $\mathbb{Z}[t]$. This approach allows one to solve various algorithmic problems for $F^{\mathbb{Z}[t]}$ using the standard Nielsen cancellation argument for the length function $L : F^{\mathbb{Z}[t]} \to \mathbb{Z}[t]$. The concept can be generalized to A-free groups for arbitrary discrete ordered abelian group A, by considering "words", indexed by elements of A rather than integers, and defining a suitable notion of reduced word. As the next paper, by Ian Chiswell, shows, the concept of an A-free group has a nice geometric interpretation: A-free groups are exactly tree free groups, that is, groups which act by isometries on some Λ-tree freely and without inversions.

Two major papers by Kharlampovich and Myasnikov develop the machinery for the study of the elementary theory of a free group. It appears that the methods are going far beyond the free groups. They provide structural and algorithmic results for a wide class of "free-like" groups, in particular, for finitely generated fully residually free groups. The theory is highly complex and it will take time to fully assess all its aspects and implications.

The authors divide their approach to elementary theories of groups and related problems into three stages. The first stage concerns equations over a given group G which is free or close to being free. In the classical terms the main problem here is to describe (effectively) the structure of solution sets of arbitrary systems of equations in finitely many variables over G. Different such descriptions are contained in the first paper. This requires the development of a fair amount of algebraic geometry over the group G and related groups (introduce algebraic sets and Zariski topology, coordinate groups and radicals, study Noetherian properties and irreducible components, to prove Nullstellensatz, etc.). On the group-theoretic level one needs to describe the algebraic structure of the coordinate groups of the irreducible varieties over G (which also appear as fully residually G groups, or limit groups (Sela), or models of the universal theory of G (Remeslennikov)). One of the ways to obtain this description is to describe effectively some canonical decomposition of such a group, so-called JSJ decomposition (introduced by Rips and Sela for finitely presented groups); this is the aim of the first paper. Algorithmically, everything is based on the so-called *elimination process*, described in the first paper; it resembles the classical elimination procedure in algebraic geometry. This process effectively relates all different techniques to each other. In the case of free groups it appears in the various forms of the Makanin-Razborov machine. In the second stage one has to introduce the main technical tool which allows one to eliminate a quantifier in a particular situation which can be described by an implicit function theorem or, in an algebro-geometric form, as lifting solutions of equations into generic points of varieties, or, in model-theoretic terms, as introducing basic Skolem's functions. Effective versions of these theorems are the target of the second paper. Implicit function theorems give the main tool to organize the *verification process* which checks whether a given formula in the group language holds in G or not. The termination mechanism, which ensures that the verification process terminates in finitely many steps, is the third stage—but these results are not included in the present volume.

The last paper of the present volume, by Esyp, Kazatchkov and Remeslennikov, studies the so-called *free partially commutative groups*. They arise naturally in many branches of mathematics and computer science, which led to a variety of names under which they are known in the literature: *semifree groups*, *graph groups*, *right-angled Artin groups*. The paper is concerned with the divisibility theory and the complexity of the word and the conjugacy problem in the partially commutative groups.

Alexandre Borovik
December 2004

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This volume reflects the major themes of the American Mathematical Society/Association for Symbolic Logic Joint Special Session (Baltimore, MD), Interactions between Logic, Group Theory, and Computer Science. Included are papers devoted to the development of techniques used for the interaction of group theory and logic. It is suitable for graduate students and researchers interested in algorithmic and combinatorial group theory.

A complement to this work is Volume 349 in the AMS series, Contemporary Mathematics, *Computational and Experimental Group Theory*, which arose from the same meeting and concentrates on the interaction of group theory and computer science.

ISBN 0-8218-3618-8

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CONM/378