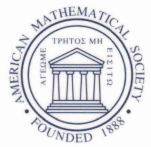
CONTEMPORARY MATHEMATICS

380

Logic and Its Applications

Andreas Blass
Yi Zhang
Editors



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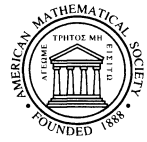

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Preface

The history of mathematical logic abounds with connections to other areas of mathematics as well as other fields like philosophy. Set theory began when Cantor needed transfinite processes for his analysis of trigonometric series. The basic notions of computability theory are needed in the study of topics ranging from Hilbert's tenth problem (in number theory) to the word problem for groups (in algebra but motivated by topology). Model theory is the foundation of modern infinitesimal analysis. Proof theory elucidates fundamental questions of epistemology.

Like most fields of mathematics, logic has, in addition to its external connections, a rich internal development, motivated by questions arising from within the field itself. Examples include forcing, large cardinals, and inner models in set theory; stability in model theory; functional interpretations and ordinal analysis in proof theory; and the priority method in computability theory. It seems fair to say that, during the last half of the twentieth century, research in mathematical logic was concerned more with such internal issues than with outreach to the rest of mathematics.

Recently, however, outreach has become stronger. The proof theorists' functional interpretations have developed to the point where they can give explicit estimates in analysis, sometimes better than what was obtained by analytic methods. Descriptive set theory, especially the branch that deals with the complexity of equivalence relations, has established connections with fields ranging from abelian group theory to ergodic theory. Technical constructions in model theory have been shown to have fascinating connections with Schanuel's conjecture in transcendental number theory. Computably enumerable sets have infiltrated Riemannian geometry.

The present volume arose from two back-to-back conferences held at the University of Michigan in April, 2003. The first, on "Logic and Its Applications in Algebra and Geometry," sought to bring together logicians (particularly model theorists and set theorists) working in these areas. The second, a workshop on "Combinatorial Set Theory, Excellent Classes, and Schanuel Conjecture," put more emphasis on pure logic, though, as suggested by the presence of Schanuel's conjecture in the title, connections with other areas were present as well.

Yi Zhang, who works on set-theoretic questions in group theory, spent the academic year 2002–2003 in my department at the University of Michigan. Shortly after he arrived, he asked me whether he could organize a conference here. I pointed out that we had no money for that. My comment turned out to be irrelevant; he volunteered to organize a conference anyway. I'm not sure how one gets prominent logicians to travel to a conference when no financial support is available, but Yi

did it. Actually, he managed to get financial support, from existing NSF grants in number theory (thanks to Hugh Montgomery and Trevor Wooley) and set theory, for one invited speaker and one graduate student, but all the other participants came at their own expense.

This seems to be the proper place to record gratitude to the NSF for the money just mentioned; to the Mathematics Department of the University of Michigan for inviting (through colloquium chairman Lizhen Ji) several of the conference speakers to give colloquium talks, for providing meeting rooms, and for supplying food for a reception; to Peter Hinman for providing excellent wine from his renowned cellar for the reception; to John Baldwin and Rami Grossberg for their help in organizing the workshop; to Bart Kastermans for setting up the conference web site and for other technical help; to the participants for making the meeting a great success; to the authors of the present papers for writing up their work for us; and to the American Mathematical Society for agreeing to publish this volume.

Andreas Blass

Two conferences, Logic and Its Applications in Algebra and Geometry and Combinatorial Set Theory, Excellent Classes, and Schanuel Conjecture, were held at the University of Michigan (Ann Arbor). These events brought together model theorists and set theorists working in these areas. This volume is the result of those meetings. It is suitable for graduate students and researchers working in mathematical logic.

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