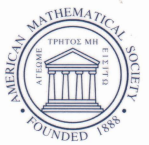
CONTEMPORARY MATHEMATICS

391

Noncommutative Geometry and Representation Theory in Mathematical Physics

Satellite Conference to the Fourth European Congress of Mathematics
July 5–10, 2004
Karlstad University, Karlstad, Sweden

Jürgen Fuchs
Jouko Mickelsson
Grigori Rozenblioum
Alexander Stolin
Anders Westerberg
Editors



American Mathematical Society

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Contents

Preface

Mathematics provides us with a language in which to formulate the laws that govern the phenomena observed in nature. This language has proven to be both powerful and effective, and, the question of how reasonable this effectiveness may be aside, in trying to understand the basic laws of nature one is bound to use the tools that mathematics supplies. A foundation of physics cannot be built solely on this ground, however; an even more essential ingredient is experiment, and any substantial progress in physics should eventually allow for predictions that can be tested experimentally. Nevertheless, the quest for a deeper understanding of fundamental physical issues, such as the interactions among elementary particles or the structure of space-time, tends to lead to theories which are ever harder to put to observational tests. In this situation, mathematical conciseness and internal consistency of a physical theory become increasingly important guidelines in the evolution of physics.

It is less evident what parts of mathematics are most relevant for the study of some given area of physics and whether or not the existing mathematical knowledge is already sufficient for addressing all problems within the area in question. In recent years, novel questions have emerged in mathematical physics, notably in quantum field theory. Accordingly, additional areas of mathematics have become influential and, in turn, been influenced themselves by the developments in physics. As a consequence, over the last two decades interactions between mathematicians and physicists have increased enormously, resulting in a fruitful cross-fertilization between different communities.

A central theme of this endeavour has been the algebraization of physical concepts. This principle manifests itself in several guises, notably in the form of non-commutative geometry and of representation theory. Each of these two areas may be discussed in its own right, but indeed they are intimately related, e.g. via the study of 'non-classical' algebraic structures which generalize the algebras of functions on a manifold. Arguably, the following list, while not being exhaustive, covers a major part of the most prominent topics encountered in these fields and in their application to the algebraization programme: quantum symmetries and quantum statistics; quantum groups, C^*-algebras, infinite-dimensional Lie algebras, vertex operator algebras, weak Hopf algebras, Lie superalgebras, Jordan triple systems; representation categories of those structures; Lie bialgebras, Gerstenhaber algebras, Frobenius and Hopf-Galois extensions; pseudodifferential operators on quantum spaces, non-commutative index theorems, moduli spaces in non-commutative geometry, operators on singular manifolds; deformation quantization, quantization of Poisson brackets, star products, Poisson-Lie groups, q-deformed and non-commutative Lagrangian field theory; modular tensor categories and the Verlinde

conjecture, Yang-Baxter equations, Calogero-Sutherland models, strings and membranes, conformal and topological quantum field theory.

Each of the topics in this list was represented by a leading expert at the international symposium *Non-commutative Geometry and Representation Theory in Mathematical Physics*. This meeting took place in July 2004 at Karlstad University in Karlstad, the capital of the Swedish province Värmland, as a satellite conference to the Fourth European Congress of Mathematics. The present volume collects contributions from most of the plenary speakers and from some of the speakers in the parallel sessions.

Browsing the table of contents or, for that matter, just the subject classification codes, the reader will undoubtedly notice that the scope of the symposium and of this volume is rather broad and that various different scientific communities are involved. Still, a detailed inspection will exhibit many interrelations between the various contributions and demonstrate that they can be regarded as different facets of a common theme. This is indeed what we experienced during the symposium ourselves and has been confirmed by many participants in their feedback after the meeting. We hope that this volume will enrich the future development in the areas to which it is devoted and that it will help to further intensify the prosperous interaction between mathematics and physics.

The 90 participants of the conference came from 25 different countries; about one third of them were from Sweden. The program consisted of 23 plenary talks and 39 presentations in parallel sessions. The schedule, abstracts of all talks and further details are available at the web site `http://www.ingvet.kau.se/teofys/conf/ncg-rt/`.

Financial support from several different sources allowed us to arrange the conference. Major contributions came from the Wenner-Gren Foundations, Längmanska kulturfonden, the Faculty Board of Karlstad University, and Karlstads kommun. Further support from Birkhäuser Publishing Inc. and from Journal of Physics A is also gratefully acknowledged. In addition, Olav Arnfinn Laudal generously provided funding for the support of several junior participants.

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We would also like to thank all the participants, the speakers and the authors of the contributions to this volume, as well as our colleagues who agreed to act as referees and devoted much time to the reviewing process.

Karlstad, June 2005

Jürgen Fuchs (Karlstad)
Jouko Mickelsson (Stockholm / Helsinki)
Grigori Rozenblioum (Göteborg)
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List of Participants

Johan Anderson
Chalmers Tekniska Högskola
Göteborg, Sweden

Nadezda Bazunova
Tallinn Technical University
Tallinn, Estonia

Niclas Bernhoff
Karlstads universitet
Karlstad, Sweden

Jonas Björnsson
Karlstads universitet
Karlstad, Sweden

Christian Blohmann
International University Bremen
Bremen, Germany

Igor Bogolubsky
Joint Institute for Nuclear Research
Dubna, Russia

Martin Cederwall
Chalmers / Göteborgs universitet
Göteborg, Sweden

Michael Chesterman
Karlstads universitet
Karlstad, Sweden

Chryssomalis Chryssomalakos
Instituto de Ciencias Nucleares UNAM
Mexico City, Mexico

Alexei Davydov
Macquarie University
Sydney, Australia

Torsten Ekedahl
Stockholms universitet
Stockholm, Sweden

Ludwig Faddeev
Steklov Mathematical Institute
St. Petersburg, Russia

Gaetano Fiore
Universitá di Napoli
Napoli, Italy

Jens Fjelstad
Universität Hamburg
Hamburg, Germany

Yaakov Friedman
Jerusalem College of Technology
Jerusalem, Israel

Anders Frisk
Uppsala universitet
Uppsala, Sweden

Jürgen Fuchs
Karlstads universitet
Karlstad, Sweden

Beatriz Gato-Rivera
Instituto de Matemáticas y Física
Fundamental
Madrid, Spain

Frank Göhmann
Bergische Universität Wuppertal
Wuppertal, Germany

Maria Gorelik
Weizmann Institute of Science
Rehovot, Israel

Nikolaj Gromov
IMM UrD RAS
Syktyvkar, Russia

Alzbeta Hakova
Silesian University
Opava, Czech Republic

Martin Hallnäs
Royal Institute of Technology (KTH)
Stockholm, Sweden

Jonas Hartwig
Chalmers Tekniska Högskola
Göteborg, Sweden

Yi-Zhi Huang
Rutgers University
Piscataway, NJ, USA

Stephen Hwang
Karlstads universitet
Karlstad, Sweden

Hans Plesner Jakobsen
Copenhagen University
Copenhagen, Denmark

Lars Kadison
University of New Hampshire
Durham, NH, USA

Noriaki Kamiya
Aizu University
Aizu-Wakamatsu, Japan

Issai Kantor
Lunds universitet
Lund, Sweden

Eugene Karolinsky
Kharkov National University
Kharkov, Ukraine

Arkady L. Kholodenko
Clemson University
Clemson, SC, USA

Vladimir V. Kisil
University of Leeds
Leeds, UK

Liivi Kluge
Tallinn Technical University
Tallinn, Estonia

Torbjörn Kolsrud
Royal Institute of Technology (KTH)
Stockholm, Sweden

Tomas Kopf
Silesian University
Opava, Czech Republic

Christian Korff
University of Edinburgh
Edinburgh, UK

Vladimir Kornyak
Joint Institute for Nuclear Research
Dubna, Russia

Petr Kulish
Steklov Mathematical Institute
St. Petersburg, Russia

Pär Kurlberg
Chalmers Tekniska Högskola
Göteborg, Sweden

Edwin Langmann
Royal Institute of Technology (KTH)
Stockholm, Sweden

Olav Arnfinn Laudal
Oslo University
Oslo, Norway

Aldo Lazar
Tel Aviv University
Tel Aviv, Israel

Dmitry Lebedev
ITEP
Moscow, Russia

Juha Loikkanen
Royal Institute of Technology (KTH)
Stockholm, Sweden

Jerzy Lukierski
University of Wrocław
Wrocław, Poland

Wladyslaw Majewski
University of Gdansk
Gdansk, Poland

Jouko Mickelsson
Royal Institute of Technology (KTH)
Stockholm, Sweden

Liesbeth Mol
Springer Science
Amsterdam, The Netherlands

Vladimir Nazaikinskii
Universität Potsdam
Potsdam, Germany

Antti Niemi
Uppsala universitet
Uppsala, Sweden

Eugen Paal
Tallinn Technical University
Tallinn, Estonia

Cornelius Paufler
Royal Institute of Technology (KTH)
Stockholm, Sweden

Juha-Pekka Pellonpää
University of Turku
Turku, Finland

Iulia Pop
Göteborgs universitet
Göteborg, Sweden

Peeter Puusemp
Tallinn Technical University
Tallinn, Estonia

Stefan Rauch-Wojciechowski
Linköpings universitet
Linköping, Sweden

Nicolai Reshetikhin
University of California
Berkeley, CA, USA

Alexander Retakh
MIT
Cambridge, MA, USA

Hjalmar Rosengren
Chalmers Tekniska Högskola
Göteborg, Sweden

Grigori Rozenblioum
Chalmers / Göteborgs universitet
Göteborg, Sweden

Maxim Samsonov
Concordia University
Montreal, Canada

Tzvi Scarr
Jerusalem College of Technology
Jerusalem, Israel

Bert Schellekens
NIKHEF
Amsterdam, The Netherlands

Gil Schieber
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro, Brazil

Urs Schreiber
Universität Duisburg-Essen
Essen, Germany

Christoph Schweigert
Universität Hamburg
Hamburg, Germany

Henrik Seppänen
Chalmers Tekniska Högskola
Göteborg, Sweden

Oleg Sheinman
Steklov Mathematical Institute
Moscow, Russia

Boris Shoikhet
ETH Zürich
Zürich, Switzerland

Sergei Silvestrov
Lunds universitet
Lund, Sweden

Eric Simring
Columbia University
New York, NY, USA

Luca Stefanini
Universität Zürich
Zürich, Switzerland

Daniel Sternheimer
Université de Bourgogne
Dijon, France

Alexander Stolin
Chalmers / Göteborgs universitet
Göteborg, Sweden

Kornél Szlachányi
KFKI Research Institute for Particle
and Nuclear Physics
Budapest, Hungary

Vitaly Tarasov
Steklov Mathematical Institute
St. Petersburg, Russia

Valeriy Tolstoy
Moscow State University
Moscow, Russia

Francesco Toppan
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro, Brazil

Osman Teoman Turgut
Bogazici University
Istanbul, Turkey

Lyudmila Turowska
Chalmers / Göteborgs universitet
Göteborg, Sweden

Leonid Vaksman
Institute for Low Temperature
Kharkov, Ukraine

Alexander Varchenko
University of North Carolina
Chapel Hill, NC, USA

Mirela Vinerean
Karlstads universitet
Karlstad, Sweden

Liam Wagner
University of Queensland
Brisbane, Australia

Benjamin Webster
University of California
Berkeley, CA, USA

Anders Westerberg
Karlstads universitet
Karlstad, Sweden

Albrecht Wurtz
Karlstads universitet
Karlstad, Sweden

Milen Yakimov
University of California
Santa Barbara, CA, USA

Genkai Zhang
Chalmers / Göteborgs universitet
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In recent years, many novel questions have emerged in mathematical physics, particularly in quantum field theory. Indeed, several areas of mathematics have lately become increasingly influential in physics and, in turn, have become influenced by developments in physics. Over the last two decades, interactions between mathematicians and physicists have increased enormously and have resulted in a fruitful cross-fertilization of the two communities.

This volume contains the plenary talks from the international symposium on Noncommutative Geometry and Representation Theory in Mathematical Physics held at Karlstad University (Sweden) as a satellite conference to the Fourth European Congress of Mathematics.

The scope of the volume is large and its content is relevant to various scientific communities interested in noncommutative geometry and representation theory. It offers a comprehensive view of the state of affairs for these two branches of mathematical physics. The book is suitable for graduate students and researchers interested in mathematical physics.

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