Bergman Spaces and Related Topics in Complex Analysis
Proceedings of a Conference in Honor of Boris Korenblum's 80th Birthday

November 20–22, 2003
Barcelona, Spain

Alexander Borichev
Håkan Hedenmalm
Kehe Zhu
Editors
Bergman Spaces and Related Topics in Complex Analysis
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   SERGUEI SHIMORIN

A Sharp Norm Estimate of the Bergman Projection on $L^p$ Spaces
   KEHE ZHU
Preface

This volume contains fourteen research papers on Bergman spaces and related topics. It grew out of a conference in honor of Boris Korenblum's 80th birthday, held on November 20–22, 2003, at the University of Barcelona, organized by J. Bruna, H. Hedenmalm, B. Pinchuk, K. Seip, and K. Zhu. On a personal note, we would like to say that we appreciate Boris Korenblum’s deep research contributions to Complex Analysis, as well as the generous support he has extended to colleagues, PhD students, and post-docs.

The volume includes also the list of participants, the list of lectures, and the opening words delivered by Kristian Seip on November 20, 2003, welcoming Boris Korenblum.

Finally, we add the list of publications of Boris Korenblum.

We thank the authors for their contributions and the referees for their careful and patient work.

Alexander Borichev
Håkan Hedenmalm
Kehe Zhu
A Word from the Organizers

The conference honoring Boris Korenblum was made possible by generous support from the following sources:

- Departament de Matemàtiques, Universitat Autònoma de Barcelona
- Facultat de Matemàtiques, Universitat de Barcelona
- European Research Training Network in Classical Analysis, Operator Theory, Geometry of Banach spaces, their interplay and their applications (contract No. HPRN-CT-2000-00116)
- Gelbart Research Institute for the Mathematical Sciences at Bar-Ilan University.

We thank these organizations for making the conference possible. We also thank the city of Barcelona for offering a pleasant background.

Joaquim Bruna
Håkan Hedenmalm
 Bernard Pinchuk
 Kristian Seip
 Kehe Zhu
List of Participants

Abakumov, Evgeny
University Marne-la-Vallee

Abuzyarova, Natalia
KTH, Stockholm

Aleman, Alexandru
Lund University

Anderson, J. Milne
University College London

Armitage, David
Queen’s University, Belfast

Ascensi, Gerard
Universitat Autònoma de Barcelona

Atzmon, Aharon
Tel Aviv University

Bieche, Camille
Université de Provence

Blasco, Oscar
Universidad de Valencia

Blasi Babot, Daniel
Universitat Autònoma de Barcelona

Borichev, Alexander
CNRS – Université de Bordeaux I

Bruna, Joaquim
Universitat Autònoma de Barcelona

Buckley, Stephen
NUI Maynooth

Burgués, Josep M.
Universitat Autònoma de Barcelona

Cantón Pire, Álicia
Universitat Autònoma de Barcelona

Carlsson, Linus
CRM

Carmona, Joan Josep
Universitat Autònoma de Barcelona

Carro, Maria J.
Universitat de Barcelona

Cascante, Carme
Universitat de Barcelona

Cerdà, Joan
Universitat de Barcelona

Clop, Albert
Universitat Autònoma de Barcelona

Cufí, Julià
Universitat Autònoma de Barcelona

Demange, Bruno
Universitat Autònoma de Barcelona

Domanski, Pawel A.
Adam Mickiewicz University, Poznan

Donaire Benito, Juan Jesús
Universitat Autònoma de Barcelona

Dussau, Xavier
Université de Bordeaux I

Dyakonov, Konstantin
Universitat de Barcelona

Fedorovskiy, Konstantin
Moscow State University (MGU)

Fernández Arias, Arturo
Universidad Nacional de Educación a Distancia

Gallardo, Eva A.
Universidad de Zaragoza
Gardiner, Stephen  
University College Dublin

González Fuente, María José  
Universidad de Cádiz

González Llorente, José  
Universitat Autònoma de Barcelona

Granados Sanandres, Ana  
University of British Columbia

Gulisashvili, Archil  
Ohio University & CRM

Hayman, Walter K.  
Imperial College

Hedenmalm, Håkan  
KTH Stockholm

Kelley, Karim  
Université de Provence

Korenblum, Boris  
SUNY at Albany

Lemmers, Oscar  
Universitat Autònoma de Barcelona

Louhichi, Issam  
Université de Bordeaux I

Lovera, Stéphanie  
Université de Provence

Lyubarskii, Yuiri  
NTNU Trondheim

Marshall, Donald  
University of Washington

Martin Pedret, Joaquim  
Universitat Autònoma de Barcelona

Marzo Sanchez, Jordi  
Universitat de Barcelona

Massaneda, Xavier  
Universitat de Barcelona

Mateu, Joan  
Universitat Autònoma de Barcelona

Melnikov, Mark  
Universitat Autònoma de Barcelona

Monreale Galan, José Ignacio  
Universitat Autònoma de Barcelona

Montes Rodríguez, Alfonso  
Universidad de Sevilla

Nicolau, Artur  
Universitat Autònoma de Barcelona

Nikolski, Nikolai  
Université de Bordeaux I

O’Farrell, Anthony G.  
NUI Maynooth

Olofsson, Anders  
KTH Stockholm

Oròbitg, Joan  
Universitat Autònoma de Barcelona

Ortega, Joaquim M.  
Universitat de Barcelona

Pascuas, Daniel  
Universitat de Barcelona

Pau, Jordi  
Universitat Autònoma de Barcelona

Perdomo Gallipoli, Yolanda  
Lund University

Pereyra, Cristina  
University of New Mexico

Perez-Gonzalez, Fernando  
Universidad de La Laguna

Pinchuk, Bernard  
Bar-Ilan University and Netanya Academic College

Ponce Escudero, Manuel  
Universidad de Sevilla

Prat Baiget, Laura  
Universitat de Barcelona

Richter, Stefan  
University of Tennessee

Rippon, Philip J.  
Open University

Saksman, Eero  
University of Jyväskylä
Saludes, Jordi
Universitat Politècnica de Catalunya

Segura Manzano, María Dolores
Universidad de Sevilla

Seip, Kristian
NTNU Trondheim

Shapiro, Harold
KTH Stockholm

Shimorin, Serguei
KTH Stockholm

Sodin, Mikhail
Tel Aviv University

Soria, Javier
Universitat de Barcelona

Stray, Arne
Bergen University

Strouse, Elizabeth
Université de Bordeaux I

Sundberg, Carl
University of Tennessee

Thomas, Pascal J.
Université Paul Sabatier, Toulouse

Tolsa, Xavier
Universitat Autònoma de Barcelona

Verdera, Joan
Universitat Autònoma de Barcelona

Walsh, David
NUI Maynooth

Youssfi, Hassan
Université de Provence

Zalcman, Lawrence
Bar-Ilan University

Zhu, Kehe
SUNY at Albany
Conference Program

THURSDAY, NOVEMBER 20, 2003

16.45: Kristian Seip

17.00–17.50: W. Hayman

18.15–19.05: L. Zalcman

19.30–20.20: N. Nikolski

Opening speech
Domination in sets and in norm
Normal and quasinormal families of meromorphic functions
Condition numbers of multipliers and Toeplitz operators

FRIDAY, NOVEMBER 21, 2003

17.00–17.50: D. Marshall

10.30–11.20: Y. Lyubarskii

12.00–12.50: A. Nicolau

15.40–16.30: K. Zhu

17.10–18.00: S. Richter

18.40–19.30: C. Sundberg

21.30: Official Banquet

Convergence of the Zipper algorithm for conformal mapping
Interpolation in generalized Paley-Wiener spaces
Regularity of measures, entropy and the law of the iterated logarithm
The Bergman projection and related integral operators
The index of invariant subspaces in spaces of analytic functions
Nontangential limits in $L^p(\mu)$ spaces

SATURDAY, NOVEMBER 22, 2003

9.00–9.50: A. Aleman

10.30–11.20: H. Hedenmalm

12.00–12.50: S. Shimorin

15.00–16.00: Open problems

16.30–17.20: H. Shapiro

18.00–18.50: M. Sodin

19.30–20.20: A. Atzmon

A Korenblum type estimate for Moebius invariant spaces of analytic functions
Bergman spaces and differential geometry
Weighted Bergman spaces and the estimates of derivatives of univalent functions
Algebraic aspects of the Dirichlet problem
How often do analytic functions visit an angle?
Banach spaces of entire functions of zero exponential type
Opening words by Kristian Seip at the “Korenblum Fiesta”, Barcelona, November 20–22, 2003

Dear Boris and dear friends of Boris!

On behalf of the organizers, I am happy to welcome you all here, for this celebration of Boris’s 80th birthday and his contribution to our science. I think Barcelona is close to perfect for what we want this meeting to be, a rather informal gathering of close scientific and personal friends of Boris in a relaxed atmosphere, with excellent food, wine, and pleasant surroundings. I know that you, Boris, as many others in this room, have enjoyable memories from time spent here, and this certainly makes Barcelona no less appropriate as the place for this meeting. So I am very happy that Joaquim took on the job of organizing the meeting; and since I did very little myself, I dare say he has taken care of it in an excellent way.

I will not give complete review of Boris’s scientific life—we will learn more about that during the meeting. But I will mention two highlights. I believe the first may come as a surprise to you. It is from CT—computed tomography. I quote from the introductory chapter entitled “In the Beginning” by Steve Webb from the 1988 volume The Physics of Medical Imaging: “It is perhaps less known that a CT (Computed Tomography) scanner was built in Russia in 1958. Korenblyum et al. [Tetelbaum, Tyutin] (1958) published the mathematics of reconstruction from projections together with experimental data and wrote: ‘At the present time in Kiev Polytechnic Institute, we are constructing the first experimental apparatus for getting X-ray images of thin sections by the scheme described in this article’.” May I remind you that G. N. Hounsfield received the 1979 Nobel Prize for Physiology and Medicine for his construction of a machine used to X-ray computed tomography in a clinical environment. I suspect that a neutral observer may find Boris’s achievements in this field more significant than our precious theory of Bergman spaces. Let me add that Boris has kept up his interest in physics. As late as last year, he published a paper entitled “Classical Properties of Low-Dimensional Conductors: Giant Capacitance and Non-Ohmic Potential Drop,” with Emmanuel Rashba, in Physical Review Letters!

Let me make a big jump to something we all know well: the two famous Acta papers. There are probably not many people who have really penetrated all aspects of those papers. The reviewer, Walter Hayman, ended his review of the first of them in Mathematical Reviews with the following words: “The above sketch must suffice to give some idea of this extremely complicated but profound paper.” I have personally been very much inspired by those papers, which contain amazing and deep ideas. One of the most striking aspects is the way linear programming enters the study of zero sets for analytic functions. I am not able to guess how you got the necessary insight, but it is clear that it is based on a broad knowledge
and understanding. As far as I know, there is no other way of getting such precise estimates for zero sequences for functions in Bergman spaces.

Boris played a decisive role in the development of the theory of Bergman spaces since around 1990, both through his papers and as a mentor. I asked Håkan about Boris’s role as a mentor, and got the following words from him: “I think Boris is one of the truly passionate mathematicians. He really believes that Mathematics is important to the real world, and is willing to discuss it at length at any time, not just during working hours. He gave a lot of support at a time when I felt my mathematical ideas met with little or no response, and I was not sure that I wanted to continue doing mathematics. He also gave me a whole new (to me) field to study: the Bergman spaces. The maximum principle he was then working on suggested that really new phenomena could appear here. I got the idea to introduce extremal problems, in a very simple setting with a single zero at first; the buzz-word we used in these early discussions with Boris was the “envelope.” Boris was always very generous, and refused co-authorship when he felt that his contribution was not quite up to the mark he set. The work on the “envelope” gave rise to the factorization theory you now know. The use of Green’s formula and the like was stimulated by a paper of Boris, “Transformation of zero sets by contractive operators in the Bergman space,” which should be appreciated better than at present.”

The last statement is certainly interesting; I believe Håkan is right. There are probably ideas in that paper that should be pursued and that could give more insight into the zero sets of Bergman spaces. I know Boris himself has thought and still thinks about that.

Mathematics has obviously meant a lot to you, Boris, but we know there have been hardships in your life of a different caliber than most of us have experienced, such as your service as a soldier in the Red Army during World War II and your painful procedure for emigration as well as immigration to Israel and the US. Most of us know little about these sides of your life. What we know is your passion for mathematics, your generosity with ideas and willingness to help fellow mathematicians. I asked another friend of mine about Boris and got the following response: “Boris always works independently of others. He usually finds new questions that are of interest to him regardless of whether they are fashionable or not. Yet pretty often those questions happen to be the key ones in new areas.” The theory of Bergman spaces certainly is a good example in this respect.
Publications of Boris Korenblum


34. (with V. S. Korolevich) Analytic functions that are regular in a disk and smooth on its boundary, Mat. Zametki 7 (1970), 165–172.


38. The functions that are holomorphic in the disk and smooth up to its boundary, Dokl. Akad. Nauk SSSR 200 (1971), 24–27.
40. Invariant subspaces of the shift operator in a weighted Hilbert space, Mat. Sb. 89 (1972), 110–137.
42. (with V. M. Faĭvyshevskiï) A certain class of compression operators that are connected with the divisibility of analytic functions, Ukrain. Mat. Zh. 24 (1972), 692–695.


67. (with J. E. McCarthy) Non-attainable boundary values of \(H^\infty\) functions, Extracta Math. 8 (1993), 138–141.


83. *Blaschke sets for Bergman spaces*, (this volume).
This volume grew out of a conference in honor of Boris Korenblum on the occasion of his 80th birthday, held in Barcelona, Spain, November 20–22, 2003. The book is of interest to researchers and graduate students working in the theory of spaces of analytic function, and, in particular, in the theory of Bergman spaces.