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Integral Geometry and Tomography

AMS Special Session
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Rider University
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April 17–18, 2004

Andrew Markoe
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Preface

The strong relationship between deep pure mathematics and applications is an exciting feature of integral geometry and its applied cousin tomography. This volume brings together fundamental research in these areas. It grew out of the special session, "Tomography and Integral Geometry" that met at the 997th Meeting of the American Mathematical Society at Rider University, April 17th and 18th, 2004.

Articles in these proceedings are written on spherical Radon transforms [**4, 18**], the $k-$plane transform [**25**], and transforms on Grassmannians and Stiefel manifolds [**15, 29**]. Applications to partial differential equations are included in the papers [**4, 18, 32**], and results on tomography may be found in [**7, 10, 20**]. The paper [**13**] is concerned with wavelets and exhibits the interplay between geometry and analysis typical of integral geometry. The paper [**30**] contains several interesting results including one on reconstructing functions from an averaging process.

Integral geometry divides into two major branches. Probabilistic integral geometry is concerned with the application of probability theory to geometric problems and is characterized by such results as Crofton's theorem and the Buffon needle problem. The solution to the Buffon needle problem determines the probability that a needle that is randomly dropped onto a plane with equally spaced parallel lines will intersect one of these lines. The other branch of integral geometry may be called Radon integral geometry. It investigates properties of functions that can be determined by integration over families of submanifolds of a given manifold. In more detail, this version of integral geometry is concerned with properties of functions that can be determined by a pair of integral transforms called the Radon transform and the dual Radon transform.

These transforms are intimately connected with the geometry of the ambient manifold. If it is possible to specify an incidence relation between two families of subsets A and B of the manifold, then one defines the Radon transform as the integral operator which for each $l \in B$ integrates functions over sets a in A incident to l. In all interesting cases one can define a dual incidence relation and consequently a dual Radon transform. This yields a profound interaction between the geometry and the analysis of the manifold which allows, in many cases, the determination of a function by knowing its Radon transform. The classic example is the Radon transform on lines in $\mathbb{R}^2$. In this case the Radon transform of a density function represents the attenuation of X-rays travelling along lines through the object, and the dual Radon transform corresponds to the operation of backprojection.

A practical application of this idea is X-ray computerized tomography in which the structure of a two-dimensional object can be determined by its integrals over

lines, these integrals being obtained from X-ray data. In the early 1970s, computerized tomography, commonly known as CT or CAT scanning, revolutionized the field of medical diagnosis.[1]

These ideas originated with Johann Radon's 1917 paper [**31**]. In this paper Radon showed how a suitably smooth function defined on $\mathbb{R}^2$ could be recovered by knowing its integrals over lines, the lines playing the role of the family of submanifolds. Radon also showed how this idea could be extended to families of hyperplanes and even to the case where the ambient manifold is the sphere or the hyperbolic plane. The major theme in Radon integral geometry is the reconstruction of a function from integrals or averages of functions over submanifolds. In some cases the reverse idea is useful, namely determining the geometric structure of subsets of the manifold via analysis of functions on the manifold.

This volume is concerned with Radon integral geometry and tomography. All the papers deal, in one way or another, with the determination of properties of functions by integral theoretic or measure theoretic methods, or by determining the geometric structure of subsets of the manifold from properties of functions on the manifold.

There has always been a close relation between pure mathematics and applications in tomography and integral geometry. Radon transforms were developed at the beginning of the twentieth century by P. Funk, G. Lorenz, and J. Radon. These researchers were motivated by problems in differential geometry, mathematical physics, and partial differential equations, respectively. In the 1970s the medical applications of these transforms produced breakthroughs in imaging technology that resulted in the 1979 Nobel Prize in Physiology and Medicine for the development of computerized tomography. The multifaceted nature of the subject was already apparent at its birth, and today the subject boasts substantial cross-disciplinary interactions, in both pure and applied mathematics as well as medicine, engineering, biology, physics, geosciences, and industrial testing. Therefore, we hope this volume will be of interest to a wide spectrum of researchers both in mathematics and also in other fields. It contains very recent work of some of the top researchers in the field.

1. Synopses of Contributed Papers

The papers contributed to this volume are arranged in alphabetical order. However, in this section we try to group the synopses by theme: papers about Radon and other integral transforms, papers on more applied topics, and papers related to partial differential equations (PDE).

Papers on integral transforms. The papers by Dobrescu and Ólafsson [**13**], Ehrenpreis [**15**], Markoe [**25**], and Ournycheva and Rubin [**29**] are concerned with recovering functions from their Radon transforms or with other types of integral transforms.

In "Wavelet Sets Without Groups" [**13**], Dobrescu and Ólafsson prove the existence of and construct certain orthonormal bases for $L^2_M(\mathbb{R}^n)$. Here M is a particular measurable subset of $\mathbb{R}^n$, possibly all of $\mathbb{R}^n$ and $L^2_M(\mathbb{R}^n)$ denotes the space

[1] Although this volume does not deal directly with computerized tomography, the interested reader may refer to Natterer [**26**], Herman [**19**], Natterer and Wübbling [**27**], or Epstein [**16**] for information on this subject. Markoe's book [**24**] contains a graphic description of computerized tomography.

of square integrable functions whose Fourier transforms have support in M. These bases are constructed from wavelets. A wavelet is a constant multiple of a function of the form $\varphi(dx+t)$ where φ is a fixed L^2 function, called the wavelet function, where d is an invertible linear transformation on $\mathbb{R}^n$ selected from a particular subset $\mathcal{D} \subset \mathrm{GL}(n,\mathbb{R})$, called the dilation set, and where t is an element of $\mathbb{R}^n$ selected from a particular subset $\mathcal{T}$ of $\mathbb{R}^n$, called the translation set. A wavelet basis is a family of wavelets that forms an orthonormal basis of $L^2_M(\mathbb{R}^n)$.

The paper of Dobrescu and Ólafsson is concerned with wavelets whose wavelet function is the inverse Fourier transform of a fixed measurable subset Ω of $\mathbb{R}^n$. The properties of such wavelets are closely related to geometric properties of the set Ω, in particular, spectral and tiling properties of Ω. Most existing results about such wavelets depend on the dilation set $\mathcal{D}$ being a subgroup of $\mathrm{GL}(n,\mathbb{R})$. A major contribution of this paper is to widen the class of dilations that are allowed and, in particular, allows for dilation sets that are not groups. The authors use techniques typical of integral geometry. These techniques include an interplay between group theory, geometry, and analysis. Furthermore, wavelets are a useful tool in such diverse fields as electrical engineering, harmonic analysis and the theory of fractals. In particular, wavelets have found applications in tomography including in work of Berenstein and Walnut [**8**], [**9**] and Candès and Donoho [**12**]. Therefore, the paper of Dobrescu and Ólafsson is of interest to integral geometers and tomographers.

The article "The Radon transform for functions defined on planes" [**15**] contributed by Ehrenpreis proves important range theorems for the Radon transform on Grassmannian manifolds. The planes referred to in the paper are affine planes in $\mathbb{R}^n$ and the Radon transform studied by Ehrenpreis takes as domain the Grassmannian of m-dimensional affine planes and is denoted by R^n_{ml}. If P is an m-plane, then $R^n_{ml}f\,(P)$ is computed by integrating f over l-dimensional planes satisfying the following incidence relation: if $m \geq l$, then the m-plane P is incident to the l-plane L if $P \supset L$. In the case $m < l$ the reverse inclusion defines the incidence relation. The case $m = 0, l = n-1$ is the ordinary Radon transform on $\mathbb{R}^n$. Ehrenpreis sketches the elementary theory of these transforms in the context of the treatment of Radon transforms developed in his book [**14**]. He studies range conditions for these transforms and shows that the John equations hold if and only if $m = n-1$. He then shows that when $m > 0$ the 0-th moment of R^n_{ml} is not independent of the base plane and hence even the moment condition of order 0 is not satisfied for this type of Radon transform. Ehrenpreis challenges the reader with three open problems including the question of whether R^n_{ml} is injective in general.

The k-plane transform is the analogue of the Radon transform in which one integrates functions over k-dimensional affine spaces rather than hyperplanes. It is also known as the k-dimensional Radon transform. The paper [**25**] contributed by Markoe concerns the relationship between the k-plane transform and Riesz potentials. This association has been known for a long time and there are many important and deep results in this area. However, this paper uses only the most elementary properties of the k-plane transform and Riesz potentials. After defining the basic concepts, the author develops a simple inversion formula for the k-plane transform that applies to certain L^1 functions and that is independent of the theory of Riesz potentials. As a simple corollary he derives a well-known multiplier theorem for the Riesz potentials of rapidly decreasing functions. This leads to an inversion of the Riesz potential of certain L^p functions with $p > 1$ (and ≤ 2). An

easy consequence is an inversion formula for the k-plane transform of certain L^p functions with $p > 1$. This type of result is known, but the proof here is more elementary. As a consequence a very simple proof of the classic inversion formula of Smith and Keinert [**33**], [**22**] is obtained except in the case of a single value of k when the dimension is even. Most of the paper is expository, and the author derives known results with simpler proofs. The first part of the paper is elementary in the sense that it could be read by a graduate student or non-specialist in the field who has a good background in real analysis at least through Lebesgue integration, elementary Fourier analysis on $\mathbb{R}^n$ and tempered distributions.

The classical cosine transform T acts on functions whose domain is the unit sphere in $\mathbb{R}^n$ in the following way

$$Tf(u) = \int_{S^{n-1}} f(v)\,|u \cdot v|\,dv. \tag{1}$$

Therefore the value of the cosine transform of f at a point u on the sphere is found by averaging $f(v)$ times the absolute value of the cosine of the angle between u and v as v ranges over S^{n-1}. Recent generalizations of this transform have found applications in Fourier analysis, integral geometry and Banach space theory. The generalized cosine transform admits powers of the cosine term and also allows integration over Grassmannian manifolds. The paper [**29**] by Ournycheva and Rubin develops a new approach to the theory of the generalized cosine transform. The authors introduce analytic families of cosine transforms over Stiefel manifolds that they call composite cosine transforms. These transforms are defined by replacing the inner product in (1) with composite powers of the products of the frames in the Steifel manifold. The goal is to find higher rank analogues of the inversion formula for the classical cosine transform, T, and the authors find precise conditions for injectivity (depending on the powers) and inversion formulas. Resulting applications include a refined investigation of the non-injectivity of generalized cosine transforms and a connection between zeta integrals and the composite cosine transform.

Papers related to applications. Berenstein, Gavilánez, and Baras [**7**], describe network tomography. Boman [**10**] provides an inversion method for the attenuated Radon transform with slightly more than 180° angular range. Katsevich and Zamyatin [**20**] elucidate the properties of a novel exact inversion method of Katsevich for cone beam CT with sources on a curve. Pesenson [**30**], develops sampling theory for manifolds.

Berenstein, Gavilánez, and Baras have contributed an interesting expository paper [**7**] on recent advances in network tomography on weighted graphs and on trees. The problem here is to detect disruptions to the entire network by analyzing only easily accessible points on the boundary of the network. This is of course a problem in Radon integral geometry and is the discrete analogue of electrical impedance tomography.

Boman [**10**] proves continuity of the inverse of the general attenuated Radon transform with data on slightly more than half of the circle of directions, and he proves a higher dimensional analogue. Recently, the invertibility of the attenuated Radon transform was established. This was an open problem for many years and was solved by Arbuzov, Bukhgeĭm and Kazantsev [**6**] and Novikov [**28**] with some extensions and simplifications contributed by others. Even more, Novikov [**28**] and Boman and Strömberg [**11**] were able to show that the attenuated Radon transform

is injective when restricted to a nonempty open subset Ω of the sphere S^{n-1}. Since such subsets may have arbitrarily small measure, it is unlikely that there could be a stable inversion formula for the attenuated Radon transform restricted to these subsets. In fact, Boman, in his article in these proceedings, shows that this inversion is unstable if data is only known on a set with less than half the measure of the sphere. The main result of Boman's article [**10**] is that a generalized Radon transform whose angular variable is restricted to even slightly more than half the sphere is stably invertible. The type of generalized Radon transform that he studies includes the attenuated Radon transform.

An interesting feature of Boman's article is a "short course" on the portion of the theory of pseudodifferential operators and microlocal analysis that is particularly applicable to the theory of Radon transforms. The reader who is not familiar with this subject should not miss Section 3 of Boman's paper.

Katsevich [**21**] recently developed a novel exact inversion method for cone beam X-ray tomography with sources on a curve. Remarkably, his formula has a relation with the abstract Kappa operator of Gelfand, Gindikin, and Graev [**17**], an operator that provides a framework to invert a large class of Radon transforms. In Katsevich's inversion method is a parameter, ψ, that could seem to be arbitrary. This function ψ determines a family of planes to integrate over in the filtering stage of his algorithm. The choice of ψ affects intermediate calculations in the algorithm, and it could seem that it would affect the final result. Katsevich and Zamyatin explain in their contribution to this volume, [**20**], why this choice does not change the final result of the algorithm. Their proof uses first principles rather than the final result of the inversion formula, and it helps clarify the formula itself.

"Frames for Spaces of Paley-Wiener Functions on Riemannian Manifolds" [**30**] by Pesenson is concerned with a generalization of the Paley–Wiener theorem and the Shannon sampling theorem to a large class of Riemannian manifolds. He defines a generalization of Paley-Wiener functions to Riemannian manifolds and shows how such functions can be reconstructed in a stable way from some countable sets of their inner products with certain distributions of compact support. This is the analogue of the Shannon sampling theorem. He also shows how to reconstruct f from weighted averages of $(1+\Delta)^k f$, where k ranges over the natural numbers. From this it is clear that the results of this paper are interesting and significant and are directly related to the major theme in Radon integral geometry of reconstructing functions from averaging processes.

Papers related to PDE. There has been a long association between the theory of the Radon transform and that of partial differential equations. It is known that the range of the k-dimensional Radon transform on $\mathbb{R}^n$ is determined by solutions to certain systems of partial differential equations for $k < n-1$. In the other direction, solutions to the Cauchy problem for the wave equation may be expressed as superpositions of plane waves involving derivatives of the Radon transform.

The spherical transform also has a strong relationship to the wave equation in, for example, the Poisson-Kirchoff formula. This formula gives the solution to the wave equation IVP in terms of spherical means of the initial data. Agranovsky and Quinto [**4**] and Rennie [**32**] use this connection to develop properties of stationary

sets—points in $\mathbb{R}^n$ at which the solution to the wave equation never moves. Gonzalez and Zhang [**18**] use mean-value operators to solve a modified wave equation on the sphere.

Consider the Cauchy problem

$$\Delta u(x,t) = u_{tt}, \qquad u(x,0) = 0, \qquad u_t(x,0) = f(x),\ f \in C_c(\mathbb{R}^n). \tag{2}$$

Let $T : \mathbb{R}^n \to [0,\infty)$ and define the T-stationary set $S_T[f]$ as

$$S_T[f] = \{x \in \mathbb{R}^n : u(x,t) = 0\ \forall t > T(x)\}$$

where u is the solution to (2). The (ordinary) stationary set is $S[f] = S_T[f]$ if T is identically zero. The authors establish relationships between $S_T[f]$ and $S[f]$ for general $T(x)$.

Stationary sets $S[f]$ are contained in zero sets of harmonic polynomials union lower-dimensional algebraic sets, and Agranovsky and Quinto [**2**] conjectured that stationary sets are the union of zero sets of *homogeneous* harmonic polynomials and lower-dimensional algebraic sets. They proved the conjecture in the plane [**2**], but this conjecture has not been proved in $\mathbb{R}^n$ despite much work and several partial results. Their paper [**4**] in this volume provides more evidence for the conjecture. Using microlocal techniques, they show that stationary sets in $\mathbb{R}^n$ are ruled surfaces if the initial data, f, has convex support. In recent work, Ambartsoumian and Kuchment [**5**] (which they reported in the special session) and Boman (unpublished) used PDE techniques to prove this theorem and a generalization. Agranovsky and Quinto prove their conjecture if the support of f is bounded by an ellipsoid. These results provide encouragement for the truth of the conjecture.

Rennie [**32**] investigates stationary sets for the Dirichlet problem for the wave equation on a square in the plane. Agranovsky and Quinto showed [**3**] for this case that, if the initial data are supported strictly in the interior of the square, then the only curves in the stationary set are lines intersecting the boundary at 90° or 45° angles, and Quinto had conjectured that these were the only possible angles in general. Rennie proved by numerical experiments and rigorous calculations that, in general, stationary curves can intersect the boundary in other angles, and her result strongly suggests that simple geometric proofs are unlikely for Agranovsky and Quinto's result.

Gonzalez and Zhang [**18**] give a closed-form solution of the modified wave equation on the sphere using the mean value operator. This operator is the Radon transform that integrates functions over subspheres of the sphere. The authors start with a series solution of Lax and Phillips [**23**] to this PDE and a series expression for the mean value operator in spherical harmonics. They relate these series to derive the closed-form solution of this modified wave equation. Then, they give a geometric proof using a commutation property of Abouelez and Daher [**1**] for the modified wave operator and the mean value operator. The result is a Poisson-Kirchoff formula for the modified wave equation on the sphere, and it is analogous to the mean-value formulas for the wave equation on hyperbolic and Euclidean spaces.

The editors would like to thank the American Mathematical Society for their support and help with the special session. We thank AMS Editorial Assistant Christine Thivierge for her cooperation with the editors and her able and thorough job putting the proceedings together. We thank the participants in the special

session for their clear, intriguing talks and thoughtful questions. Andrew Markoe thanks his wife Ruth and his children, Ariana, Abigail, and Emily, for their love and support and for tolerating him while he was working on these proceedings. Todd Quinto thanks his wife, Judy Larsen, and their daughter, Laura, for their love, encouragement, and support. We hope you find the articles stimulating.

Andrew Markoe and Eric Todd Quinto, December 2005

2. Talks Presented at the Special Session

There are continuing breakthroughs in tomography and integral geometry, and these were well represented at the special session. The speakers included a broad sample of the major researchers in the fields. Other researchers and students also participated in the special session.

C. Berenstein's joint work with F. Gavilánez on Internet tomography is novel and appealing, and it could have practical implications for locating problems on the Internet. J. Boman's joint work with J.- O. Strömberg provides an important simplification to Novikov's inversion formula for the attenuated Radon transform of emission tomography. The Boman-Strömberg technique also leads to a generalization of Novikov's results to a more general Radon transform. This generalization explicates the nature of attenuated Radon transforms among generalized Radon transforms. The most modern X-ray CT scanners use X-ray sources on a helix, and A. Kastevich, one of our speakers, presented some of his newest work on stability of his revolutionary algorithm.

Radar, Sonar, and thermoacoustic tomography can be modeled by spherical integrals. This was addressed by M. L. Agranovsky, and the joint work of P. Kuchment and G. Ambartsoumian. Their results provide important uniqueness and injectivity results. L. V. Rachele's work on impedance imaging is important theoretically and involves deep integral geometry of geodesic transforms. B. Rubin presented deep theorems about the Radon transform on matrix groups. F. B. Gonzalez talked about joint work with T. Kakehi on new support theorems for Radon transforms on Grassmannian manifolds. L. Ehrenpreis generalized his formalism of the parametric Radon transform to Lie groups. This could help simplify and unify proofs in this case, as it has done for the transform on $\mathbb{R}^n$. E. L. Grinberg proved theorems about Radon transforms on isotropic planes. G. Ólafsson described recent results of his on wavelets. E. T. Quinto gave new mean value extension theorems on manifolds. I. Pesenson talked about average variational splines on weighted graphs. S. Alesker described his results on valuations. E. M. Stein proved deep theorems for Radon transforms on the Heisenberg Group.

Here is the complete list of talks. The title of each talk is listed, followed by the author(s) and a reference to the abstract number. In the case of joint authors, the speaker is denoted by an asterisk. Abstracts may be found in *Abstracts of Papers Presented to the American Mathematical Society*, Volume 25, Issue 3, 2004.

- Reconstruction on Spheres
  - Mark L. Agranovsky, Bar-Ilan University
  - (997-45-140)
- Theory of Valuations and Integral Geometry
  - Semyon Alesker, Tel Aviv University
  - (997-52-59)

- The Circular Radon Transform and Thermo-Acoustic Tomography
  - Peter Kuchment*, Texas A&M University
  - Gaik Ambartsoumian, Texas A&M University
  - (997-44-219)
- Internet Tomography
  - Carlos A. Berenstein*, University of Maryland
  - Franklin Gavilánez, University of Maryland
  - (997-44-82)
- Novikov's Formula for the Attenuated Radon Transform – a New Approach
  - Jan Boman*, Stockholm University
  - Jan-Olov Strömberg, KTH, Stockholm
  - (997-44-110)
- The Radon Transform and PDE
  - Leon Ehrenpreis, Temple University
  - (997-44-101)
- Moment Conditions and Support Theorems for Radon Transforms on Affine Grassmannians
  - Fulton B Gonzalez*, Tufts University
  - Tomoyuki Kakehi, University of Tsukuba
  - (997-44-69)
- Radon Transforms on Isotropic Planes
  - Eric L. Grinberg, University of New Hampshire
  - (997-44-164)
- Stability Estimates for Helical Computed Tomography
  - Alexander Katsevich, University of Central Florida
  - (997-44-171)
- Groups, Wavelets, and Harmonic Analysis on R^n
  - Gestur Ólafsson, Louisiana State University
  - (997-42-122)
- Interpolation by Average Variational Splines on Weighted Graphs
  - Isaac Pesenson, Temple University
  - (997-41-108)
- Mean Value Extension Theorems and Microlocal Analysis
  - Eric Todd Quinto, Tufts University
  - (997-44-21)
- The Ray Transform and Inverse Problems for Elastic Media
  - Lizabeth V. Rachele, University at Albany, SUNY
  - (997-35-145)
- Radon Transform on Matrix Spaces
  - Boris Rubin, The Hebrew University of Jerusalem
  - (997-44-23)
- Radon Transforms, the Heisenberg Group, and Discrete Analogues
  - Elias M. Stein, Princeton University
  - (997-26-107)

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