Recent Developments in Algebraic Topology

A Conference to Celebrate Sam Gitler's 70th Birthday
December 3–6, 2003
San Miguel de Allende, México

Alejandro Ádem
Jesús González
Guillermo Pastor
Editors

American Mathematical Society
Recent Developments in Algebraic Topology
Recent Developments in Algebraic Topology

A Conference to Celebrate Sam Gitler's 70th Birthday
December 3–6, 2003
San Miguel de Allende, México

Alejandro Ádem
Jesús González
Guillermo Pastor
Editors

American Mathematical Society
Providence, Rhode Island
Contents

Preface vii

A personal appreciation of Sam Gitler ix
E. MICHA

List of Participants xv

The mathematical work of Sam Gitler, 1960–2003 1
D. M. DAVIS

The topology of the category of open and closed strings 11
N. A. BAAS, R. L. COHEN, and A. RAMÍREZ

A spectral sequence approach to normal forms 27
M. BENDERSKY and R. C. CHURCHILL

On the degree 2 map for a sphere 83
F. R. COHEN and I. JOHNSON

On the fibrewise Poincaré-Hopf theorem 101
C. L. DOUGLAS

A localization principle for orbifold theories 113
T. de FERNEX, E. LUPERCIO, T. NEVINS, and B. URIBE

On classifying spaces for the family of virtually cyclic subgroups 135
D. JUAN-PINEDA and I. J. LEARY

Symmetric products of two dimensional complexes 147
S. KALLEL and P. SALVATORE

Block bundle obstruction to Kervaire invariant one 163
K. Y. LAM and D. RANDALL

Upper bounds for the span of projective Stiefel manifolds 173
P. SANKARAN and P. ZVENGROWSKI

On $\mathbb{Z}_2$-equivariant loop spaces 183
M. A. XICOTÉNÇATL
Preface

A conference on homotopy theory was held at San Miguel de Allende, México, on December 3-6, 2003. The conference covered several areas of current interest in the field as can be seen from the papers in this volume.

The meeting was organized by the Department of Mathematics of the Centro de Investigación y de Estudios Avanzados del IPN (CINVESTAV) to celebrate Sam Gitler’s seventieth birthday. It was extraordinary to see that the meeting was attended by many of his collaborators, colleagues, students, and friends.

Sam Gitler was born in México City in 1933. He met Solomon Lefschetz at the National University of México while he was doing his undergraduate studies. After graduating from engineering in 1956, he arrived at Princeton University dreaming about studying algebraic geometry. His interests, however, moved him rapidly towards algebraic topology, and he obtained his Ph.D. in 1960 under the supervision of Norman Steenrod.

After a short period at Brandeis University, Sam was invited in 1961 by José Ádem to start the Mathematics Department of the newly created Centro de Investigación y de Estudios Avanzados del IPN. Ádem certainly made a wise decision, as during those early years both created the right atmosphere to develop for the first time in México extraordinary mathematical activity. He succeeded José Ádem as Chairman of the Department from 1973 to 1981 and seven years later was appointed Chairman of the Mathematics Department at the University of Rochester. Sam joined the Mathematics Department of the Centro de Investigación again in 2000 after retiring from Rochester.

In spite of the different important positions he has held, he has always been an enthusiastic topologist in his more than forty years of professional activity. Sam has written over forty papers as well as several books. All of them show a deep knowledge of topology and an acute point of view.

Sam has also been an active teacher. Several generations of mathematicians have benefited from his expertise as well as from his immense generosity.

Sam was appointed in 1986 as a member of El Colegio Nacional, the highest recognition offered by the Mexican scientific and artistic community.

It is indeed a privilege to have Sam back in México working with the same enthusiasm as ever.

The meeting was possible due to the generous financial support of Centro de Investigación y de Estudios Avanzados del IPN as well as of Consejo Nacional de Ciencia y Tecnología.
PREFACE

It is a pleasure to thank all participants and contributors for their enthusiasm and to offer Sam this volume.

Alejandro Ádem
Jesús González
Guillermo Pastor
A personal appreciation of Sam Gitler

Elias Micha

It is not possible to write about Sam Gitler without first expressing the great honour that this means for me. Notwithstanding that he is an imposing mathematical figure, his immense passion, generous smile, amiability and lightheartedness have made my life so much better only for having had the chance to meet him and share it with him.

When I first reflected upon the fact that we were going to organize a conference to celebrate Sam’s seventieth birthday, I thought that something was amiss, because although these celebrations are intended for people with a great trajectory, generally they are retired and in the declining years of their academic life. Nothing less true than this in the case of Sam. His energy, both physical and intellectual, is inexhaustible and he continues doing mathematics with the same passion, zeal, and restlessness that surprised me thirty years ago.

My first encounter with Sam Gitler took place in 1975, and I recall it as an image that has prevailed in my memory ever since. I was in my third year as an undergraduate student of mathematics at the National University of Mexico and I had heard that one of the best Mexican mathematicians was an individual whose name was Samuel Gitler and that he worked at the Mathematics Department of CINVESTAV. At that time, I used to go regularly to a sports club in Mexico City before going to school. One day, when I finished working out, I entered the steam room, and instead of the usual chat about soccer and finances, I overheard a conversation about embedding manifolds in euclidean space, spiced with exotic spheres and algebraic invariants. Incredulously, I turned to the source of the conversation and saw a happy and smiling figure who was wearing foggy glasses, and who, wholly unaware of that fact, continued talking enthusiastically with his interlocutor who, I learned in the end, was Wolf Iberkleid. Immediately I thought about how to approach him and I decided to leave the steam room and wait in the showers. When I saw him again I didn’t have the courage to talk to him but in the end I dared. I approached him and said “Are you Professor Samuel Gitler?” He answered, “Yes. Who are you?” I told him that I was a mathematics undergraduate student, and immediately realized that my answer had been a big mistake, for his eyes sparkled and he started talking about mathematics as if I were a colleague and not an undergraduate student who had taken just a few courses and had hardly passed them. My next big mistake was to comment that I was attending a course on Knot Theory with “Fico” (Professor Francisco González Acuña), and our first encounter concluded with an explanation of a proof of the Van Kampen theorem which Sam devised on the spot, and of which I didn’t understand even the first word. At that time, Sam was in the prime of his mathematical activity and had been a world class
topologist for fifteen years. I can clearly visualize Sam during those years prior to our first encounter, thinking deeply about very difficult problems like the Immer- sion Conjecture and displaying an overflowing mathematical activity whose only injured party was his marvellous wife Rachel, of whom I will say more later. For now, it suffices to say that without the decisive collaboration of this extraordinary woman, many of Sam’s theorems would not have been completed.

My second encounter with Sam occurred a few months after the first one and, of course, as it implicates Sam it certainly turned out to be as peculiar as the first. It was during my last year as an undergraduate that I decided that I wanted to be a graduate student at the University of Oxford. In view of this, Sevín Recillas, one of my professors at the University of whom I was very fond of, told me, “If you want to go to Oxford you should talk to Samuel Gitler and ask him to recommend you”.

I tried to have another “casual” encounter with Sam, so I attended the sports club much more frequently, with the hope that it would occur. Unfortunately, it was not so and several weeks passed until I decided to arm myself with courage, and one day in a complete state of panic I showed up at his office in CINVESTAV. I knocked on his door and heard the indication “come in”. My recollection of that occasion is of Sam standing in front of the blackboard talking to a mathematician, possessed by his customary passion, with the distinctive everyday smile and vivacious eyes, absorbed in some mathematical problem. As it always happens in such instances, he continued thinking, and of course, he became aware that I was present only after several minutes elapsed.

In the case of Sam, the concentration in mathematics is total. It does not matter if a young man shaking with fear is standing in front of him. He abandons the real world and his mind travels to the mathematical world. He investigates, concludes, discovers and links concepts without being perturbed at all by the surrounding circumstances. Now, after thirty years and countless sessions working with him in his office, I witness the same ritual, to which I am now accustomed when somebody knocks at his door. And, since being absorbed in thought and becoming speechless are contagious, I often find myself doing the same thing.

Back to the real world, when he became aware of me and without remembering who I was, he said mechanically, “What can I do for you?” To which I replied, “I just want to talk to you about Oxford”. “Ah! You are here to ask for a recommendation, right?” In that instant I perceived for the first time the great generosity and professionalism of Sam. I managed to stammer, “Yes”. He continued, “Ah! What a coincidence! It just happens that Professor John Hubbuck present here is from the University of Oxford, and I am sure that he will join me now in an interview with you so that, depending on its outcome, we can both recommend you”. I felt that the world came to an end and I regreted having gone to Sam’s office. To be examined by two world-class mathematicians without previous notice or preparation was terrifying. The interview went by, and after a gruelling session, I left his office with the feeling of failure. Nevertheless, in September of 1976 I arrived in Oxford to start my graduate work.

Being in Oxford, again circumstances put me in Sam’s path. It so happened that Sam had been invited to Oxford for a stay of several months at All Souls College, starting in the fall of 1976.
My first year in Oxford was particularly difficult since I was admitted to the Master of Science programme, which at that time had a duration of one year, at the end of which a thesis had to be submitted, and an oral exam on several subjects was required. A good performance in the programme was essential to being able to stay in Oxford for the doctorate. Again, Sam was my lifeguard, and during that year on several occasions we spent many hours either at my place or in his house during which, with his characteristic generosity, he patiently explained concepts and problems in topology and geometry that I was preparing for my exam. It didn’t matter to Sam that I was only a student with no experience whatsoever in research; he talked to me for several hours each time.

From our common stay in Oxford and the close relationship of those days, we developed a great friendship that has strengthened since, and that not only holds but grows every day.

It was in Oxford where I had the opportunity to meet the Gitler family and where I became another member of it, inasmuch as my friendship with Sam was extended from the beginning to Rachel and to his children Isidoro, Miriam, and Susy. With incredulity, I became aware that the doors of Sam’s home were always open, and that it was a meeting place for several mathematicians and friends. The hospitality and cordiality with which me and my family were greeted by Rachel, her disposition to lodge us for several days and feed us splendidly, made me feel from the start as another member of that wonderful family, which I consider a fundamental part of my life.

Along these thirty years I have witnessed what Rachel means to Sam. Rachel is an admirable woman and is admired by all of us. Certainly Sam owes her a lot. During all these years I have seen the many times in which the Gitlers have changed residence: Mexico City, Oxford, Princeton, Tel Aviv, Rochester, Rio de Janeiro, Barcelona, and many more. It is fair to say that without Rachel’s disposition, all of these long or short stays, in addition to hundreds of trips to conferences that were so important to Sam’s career, would have not been possible. Rachel always made the necessary arrangements to be with her loving companion.

It is also fair to say that Sam has never packed a thing for any of these multiple trips. The only thing that I have seen him do is to stuff his briefcase hurriedly with as many books and papers as will fit, and that he expects to need because they are relevant to the problem that he is thinking about at the time he has to travel. The truth is that he never puts in the briefcase the proper ones. However, thanks to Rachel, whenever he reaches his destination he always succeeds in getting the correct papers as well as everything else he needs.

A distinctive quality of Sam’s personality that I cannot omit is his exaggerated humility. Just as much in the clothes that he wears everyday, as in his personal manner, Sam does not carry on his shoulders the weight of the postures of the famous and conceited intellectual, or of the inaccessible man of success. On the contrary, he seems to pass, anxious not to be noticed. I recall the occasion in Oxford when, at tea time, Sam was taking about mathematics (for a change) to a group of students, myself included, when he hastily told us that he was going to travel to México for about four days to receive a small recognition, and rapidly returned to the subject of mathematics. Several months later we learned that in that trip to México, Sam received from the President of the country, The National Prize for the Sciences, which is the highest award possible for a Mexican Scientist.
As in my case, Sam’s generosity has played a fundamental role in the careers of many generations of Mexican mathematicians. The list is so large that it would be impossible here to mention all those who in different ways have benefited from Sam. However, I shall mention the case of Ernesto Lupercio, one of the more successful young Mexican mathematicians. Being my M.Sc. student, I quickly realized how brilliant Ernesto was and I thought that he should not be my student because “he had to play in better leagues”. So I did not let him complete his M.Sc. as my student, and I pressed him to apply to several of the best universities in the United States. I talked to Sam about him. Sam contacted Ernesto and arranged an interview with him that lasted several hours after which Sam wrote the essential letters of recommendation and made the pertinent phone calls. The result was that Ernesto was accepted in Stanford University where he completed his Ph.D. under professor Ralph Cohen.

When I completed my D. Phil., and returned to México in January of 1982, an incident occurred, in which again Sam was the protagonist, that changed my life and established the possibility of talking to Sam everyday. I had a tenure track job offer at the Instituto de Matemáticas of the National University of México (UNAM), and in effect, as soon as I returned to México I had an interview with the chairman, where he confirmed the work at UNAM, welcomed me, showed me to my office and said, “you start work on Monday”. Again, Sam appeared in my life, and the Saturday prior to that Monday I received a phone call in which he said, “How nice that you are back in México so that you can start working at CINVESTAV”. I told him that it was impossible because I was already committed to start working on Monday at UNAM. Sam insisted, but finally said, well, just come to my house tomorrow to have a drink with me. I arrived at his house on Sunday and there was Sam with his former Ph.D. students Luis Astey and Enrique Antoniano who were already Professors at the Department of Mathematics at CINVESTAV. Sam poured the promised drink and addressing Luis and Enrique said, “Tell Dr. Micha here, where will he start working on Monday”. Indeed, I started working on Monday, not at UNAM, but at CINVESTAV. To feel so welcomed and appreciated by a figure like Sam did not allow for any doubts, and I ended up apologizing to the chairman of the Instituto de Matemáticas for going back on my word. Such as Sam had described it, the Mathematics Department at CINVESTAV was a vibrant place full of mathematical activity. It was the ideal place for a young topologist. I recall the profusion of world class topologists from all over, invited by Sam for short stays at the Department, the wonderful seminar of Luis Astey about the Atiyah-Singer Index Theorem and the magnificent talks by Alberto Verjovsky on the then recent work of Thurston on 3-manifolds. But mostly what remains in my memory and in my heart from those days were the endless mathematical conversations with my dear friend Sam.

Unfortunately, that didn’t last long, because just a few months after I was hired at CINVESTAV, Sam was appointed chairman of the Mathematics Department of the University of Rochester in New York, where he stayed 15 years. While he was chairman, the Department hired, among others, topologists of the stature of D. Ravenel and F. Cohen, thus becoming a world-class center for algebraic topology. Nevertheless, Sam continued to have a very close contact with México and with me in particular. He still had Mexican students and traveled to México frequently. He
organized seminars, gave talks, and in general participated directly in many aspects of Mexican mathematical life.

In my particular case, as had always happened, fortune put me again in contact with Sam. I managed to get a job at Vassar College in Poughkeepsie, New York, where I stayed for two years. That circumstance, situated me at a 5 hour drive away from enjoying again the company of my friend, and to absorb many of his ideas. I became a collaborator of Sam! I have been participating in joint work with a team put together by Sam with L. Astey and G. Pastor. Our first collaboration started in Rochester and was completed in México in 1991 with the rest of the team, and our most recent collaboration was published in 2005.

It is very hard to describe in few pages the many aspects that I have witnessed of a personal life that has been so rich and fulfilling. However, I hope that I have been able to convey a few of them.

Finally, I would like to say that the great audacity, as much as the intellectual courage, of Sam is immense. That, in spite of their depth, his ideas appear surprisingly simple and elegant and have influenced decisively many aspects of algebraic topology as well as his colleagues. His integrity, both as a mathematician and as a person in private life, is very hard to equate. His points of view regarding the quality of research are deep and honest.

Sam Gitler, you have been and will always be the object of my admiration. With all sincerity. Thank you very much!

January, 2006
# List of Participants

<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alejandro Ádem</td>
<td>University of Wisconsin</td>
</tr>
<tr>
<td>Marcelo Aguilar</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Egidio Barrera</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Omar Becerra</td>
<td>Escuela Superior de Física y Matemáticas, IPN</td>
</tr>
<tr>
<td>Martin Bendersky</td>
<td>Hunter College, CUNY</td>
</tr>
<tr>
<td>William Browder</td>
<td>Princeton University</td>
</tr>
<tr>
<td>Edgar Brown</td>
<td>Brandeis University</td>
</tr>
<tr>
<td>Mónica Clapp</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>José Luis Cisneros</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Frederick Cohen</td>
<td>University of Rochester</td>
</tr>
<tr>
<td>Ralph Cohen</td>
<td>Stanford University</td>
</tr>
<tr>
<td>Donald M. Davis</td>
<td>Lehigh University</td>
</tr>
<tr>
<td>Samuel Gitler</td>
<td>Departamento de Matemáticas, CINVESTAV</td>
</tr>
<tr>
<td>Jesús González</td>
<td>Departamento de Matemáticas, CINVESTAV</td>
</tr>
<tr>
<td>Francisco González-Acuña</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Rolando Jiménez</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Daniel Juan-Pineda</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Kee Y. Lam</td>
<td>University of British Columbia</td>
</tr>
<tr>
<td>Santiago López de Medrano</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Ernesto Lupercio</td>
<td>Departamento de Matemáticas, CINVESTAV</td>
</tr>
<tr>
<td>James Milgram</td>
<td>Stanford University</td>
</tr>
<tr>
<td>Haynes Miller</td>
<td>Massachusetts Institute of Technology</td>
</tr>
<tr>
<td>Jack Morava</td>
<td>Johns Hopkins University</td>
</tr>
<tr>
<td>Guillermo Moreno</td>
<td>Departamento de Matemáticas, CINVESTAV</td>
</tr>
<tr>
<td>Jacob Mostovoy</td>
<td>Instituto de Matemáticas, UNAM</td>
</tr>
<tr>
<td>Guillermo Pastor</td>
<td>Departamento de Matemáticas, ITAM</td>
</tr>
<tr>
<td>Antonio Ramírez</td>
<td>Stanford University</td>
</tr>
<tr>
<td>Duane Randall</td>
<td>Loyola University</td>
</tr>
</tbody>
</table>
PARTICIPANTS

Douglas Ravenel
University of Rochester

Eduardo Santillán Zerón
Departamento de Matemáticas,
CINVESTAV

José Seade
Instituto de Matemáticas, UNAM

Bernardo Uribe
University of Michigan

Alberto Verjovsky
Instituto de Matemáticas, UNAM

Ismar Volic
University of Virginia

Clarence Wilkerson
Purdue University

Miguel Xicoténcatl
Departamento de Matemáticas,
CINVESTAV

Peter Zvengrowski
University of Calgary
This book is an excellent illustration of the versatility of Algebraic Topology interacting with other areas in Mathematics and Physics. Topics discussed in this volume range from classical Differential Topology and Homotopy Theory (Kervaire invariant one problem) to more recent lines of research such as Topological Quantum Field Theory (string theory). Likewise, alternative viewpoints on classical problems in Global Analysis and Dynamical Systems are developed (a spectral sequence approach to normal form theory).

This collection of papers is based on talks at the conference on the occasion of Sam Gitler's 70th birthday (December, 2003). The variety of topics covered in this book reflects the many areas where Sam Gitler's contributions have had an impact.