CONTEMPORARY MATHEMATICS

408

# Inverse Problems, Multi-Scale Analysis and Effective Medium Theory

Workshop in Seoul
Inverse Problems, Multi-Scale Analysis
and Homogenization
June 22–24, 2005
Seoul National University
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Habib Ammari
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# Preface

Recent developments in inverse problems, multi-scale analysis, and effective medium theory reveal that these fields share several fundamental concepts.

Over the last 10 years or so, a considerable amount of interesting work has been dedicated to the imaging of small inclusions. To image small inclusions with a good resolution one needs to design stable and accurate algorithms for the numerical computations of solutions to partial differential equations in the presence of small-size features. It is known that small-size features cause difficulties in the numerical solution by the finite element or finite difference methods. This is because such features require refined meshes in their neighborhoods, with their attendant problems. Some of the promising techniques developed in the field of multi-scale analysis are very useful in this context.

On the other hand, the derivation of optimal bounds for the volume fraction of the small conductivity inclusions are direct analogues of the corresponding estimates for the effective conductivity matrix known in the theory of composite materials.

The main purpose of this volume is to highlight the benefits of sharing new, deep ideas among the fields of inverse problems, multi-scale analysis, and effective medium theory. It provides exposition of fresh techniques for solving inverse problems, emphasizing their connection with multi-scale analysis and the mathematical theory of composite materials.

The mathematical problems that appear in these three areas are of practical importance and pose significant challenges to pure and applied mathematicians. These problems have attracted a lot of attention of researchers lately. The methods involved come from a wide range of areas of pure and applied mathematics, ranging from potential theory to partial differential equations, to scattering theory, to complex analysis, to numerical methods. The topics of this volume are addressed from analytic, numerical, and physics perspectives.

Due to the character of its topic, this volume is of interest not only to mathematicians working in these areas, but also to physicists and engineers who could communicate with mathematicians on these issues. We envision that these proceedings will stimulate much needed progress in the directions described above.

This is the proceedings of the research conference "Workshop in Seoul: Inverse Problems, Multi-Scale Analysis, and Homogenization" held at Seoul National University, June 22-24, 2005. We believe that our objective to bring together experts in these related fields and to share their ideas was successfully achieved. This was only possible thanks to the enthusiastic participation of wonderful speakers and authors of this volume. We thank them all. We also thank the staffs at the mathematics department and RIM of Seoul National University, without whom the workshop

would not have been possible. Finally, we acknowledge the generous financial support of BK21 Mathematical Science Division, Seoul National University, CNRS - KOSEF, Hokee Min Memorial Fund, Daewoo Foundation, and KARC.

Habib Ammari and Hyeonbae Kang
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Recent developments in inverse problems, multi-scale analysis and effective medium theory reveal that these fields share several fundamental concepts. This book is the proceedings of the research conference, "Workshop in Seoul: Inverse Problems, Multi-Scale Analysis and Homogenization," held at Seoul National University, June 22–24, 2005. It highlights the benefits of sharing ideas among these areas, of merging the expertise of scientists working there, and of directing interest towards challenging issues such as imaging nanoscience and biological imaging. Contributions are written by prominent experts and are of interest to researchers and graduate students interested in partial differential equations and applications.

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