Representations of Algebraic Groups, Quantum Groups, and Lie Algebras

AMS-IMS-SIAM Joint Summer Research Conference
July 11–15, 2004
Snowbird Resort, Snowbird, Utah

Georgia Benkart
Jens C. Jantzen
Zongzhu Lin
Daniel K. Nakano
Brian J. Parshall
Editors
Representations of Algebraic Groups, Quantum Groups, and Lie Algebras
Conference Group Photo
Snowbird Resort
July 2004
This volume contains the proceedings of an AMS-IMS-SIAM Joint Summer Research Conference on Representations of Algebraic Groups, Quantum Groups, and Lie Algebras, held at the Snowbird Resort, Snowbird, UT, from July 11–15, 2004, with support from the National Science Foundation, grant DMS-9973450.

2000 Mathematics Subject Classification. Primary 05E10, 14L17, 16G20, 17Bxx, 20C08, 20Gxx.

Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Library of Congress Cataloging-in-Publication Data
AMS-IMS-SIAM Joint Summer Research Conference, Representations of Algebraic Groups, Quantum Groups, and Lie Algebras (2004 : Snowbird, Utah)
p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; v. 413)
II. Series: Contemporary mathematics (American Mathematical Society) ; v. 413.
QA176.A47 2004
512'.22—dc22 2006045952

Copying and reprinting. Material in this book may be reproduced by any means for educational and scientific purposes without fee or permission with the exception of reproduction by services that collect fees for delivery of documents and provided that the customary acknowledgment of the source is given. This consent does not extend to other kinds of copying for general distribution, for advertising or promotional purposes, or for resale. Requests for permission for commercial use of material should be addressed to the Acquisitions Department, American Mathematical Society, 201 Charles Street, Providence, Rhode Island 02904-2294, USA. Requests can also be made by e-mail to reprint-permission@ams.org.

Excluded from these provisions is material in articles for which the author holds copyright. In such cases, requests for permission to use or reprint should be addressed directly to the author(s). (Copyright ownership is indicated in the notice in the lower right-hand corner of the first page of each article.)

© 2006 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights except those granted to the United States Government.
Copyright of individual articles may revert to the public domain 28 years after publication. Contact the AMS for copyright status of individual articles.
Printed in the United States of America.
⊗ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.
Visit the AMS home page at http://www.ams.org/
Contents

Preface vii
List of Talks ix
List of Participants xi
Extensions for finite groups of Lie type II: Filtering the truncated induction functor
  CHRISTOPHER P. BENDEL, DANIEL K. NAKANO, and CORNELIUS PILLEN 1
Algebras, representations and their derived categories over finite fields
  BANGMING DENG and JIE DU 25
On localization of $\mathcal{D}$-modules
  YOSHITAKE HASHIMOTO, MASAHARU KANEDA, and DMITRIY RUMYNIN 43
Representations of reduced enveloping algebras and cells in the affine Weyl group
  J. E. HUMPHREYS 63
Nakajima’s monomials and crystal bases
  SEOK-JIN KANG, JEONG-AH KIM, and DONG-UY SHIN 73
A new Lie bialgebra structure on $sl(2,1)$
  GIZEM KARAALI 101
The Steinberg tensor product theorem for $GL(m|n)$
  JONATHAN KUJAWA 123
Cyclotomic $q$-Schur algebras and Schur-Weyl duality
  ZONGZHU LIN and HEBING RUI 133
Geometric crystals and affine crystals
  TOSHIKI NAKASHIMA 157
Self-extensions for finite symplectic groups via algebraic groups
  CORNELIUS PILLEN 173
Classification of finite dimensional simple Lie algebras in prime characteristics

ALEXANDER PREMET and HELMUT STRADE 185

From quantum groups to unitary modular tensor categories

ERIC C. ROWELL 215

A trip from representations of the Kronecker quiver to canonical bases of quantum affine algebras

JIE XIAO and GUANGLIAN ZHANG 231
Preface

Representation theory has played a central role in mathematics through its rich interplay with, and applications to, many other fields. The 2004 AMS-IMS-SIAM Joint Summer Research Conference, *Representations of Algebraic Groups, Quantum Groups, and Lie Algebras*, focused on the geometric and combinatorial aspects of the subject. New developments involving quiver representations were presented in connection with important constructions for quantum groups. Another major theme was use of methods from algebraic geometry, via derived categories, to study the representation theory of algebraic groups and Lie algebras, including Kac-Moody Lie algebras, modular restricted Lie algebras (or, more generally, finite group schemes), and Lie superalgebras.

Each morning session featured principal speakers on the designated major themes. Each afternoon, two parallel sessions allowed attendees to present talks on current research, providing a forum for junior mathematicians to communicate new developments in the area, followed by ample time for informal discussions and interaction.

The present volume brings together papers from the principal speakers and other participants on a wide variety of topics in modern representation theory. Several contributions are surveys that aim to introduce the topics to a wider audience of researchers. All of the papers were carefully refereed, and the editors express their gratitude to the anonymous referees for the high standards employed in preparing their reports.

During the conference, a banquet was held to celebrate the achievements of James E. Humphreys on the occasion of his 65th birthday. Over the last 40 years, Jim's contributions have inspired many deep insights and new developments in the representations of algebraic groups and finite groups of Lie type. In addition, his well-known books in the area have brought vast, intertwined research topics together in a concise and coherent manner. Jim has also encouraged many of us by taking a genuine interest in our work. Several months before the conference, he formally retired from the University of Massachusetts, Amherst, to devote himself to research and to book writing. We are delighted to include here one of his recent articles, which poses an interesting conjecture relating irreducible representations of semisimple Lie algebras in positive characteristics to left cells in affine Weyl groups.

Financial support for the conference was provided by a grant from the National Science Foundation, and the staff of the American Mathematical Society provided considerable logistical support. In particular, the organizers acknowledge Wayne Drady for his professional dedication to managing the conference and Christine M. Thivierge for her patience and help in editing this volume. We also thank the participants for making the conference a success: the speakers during the conference
and at the banquet and the afternoon session chairs for their work in keeping the
conference on schedule. Special thanks go to Leonard Scott, whose toastmasters
during the banquet provided many humorous and wonderful memories.

Georgia Benkart
Jens C. Jantzen
Zongzhu Lin
Daniel K. Nakano
Brian J. Parshall
January 2006
List of Talks

Talks by Principal Speakers

Henning H. Andersen,
*Cohomology of line bundles*

Jie Du,
*Strong monomial basis property and canonical basis for a cyclic quiver*

Eric M. Friedlander,
*π-points for finite group schemes*

Seok-Jin Kang,
*Crystal bases for quantum affine algebras and combinatorics of Young walls*

Alexander Kleshchev,
*On the structure of finite W-algebras of type A*

Ivan Mirković,
*Beilinson-Bernstein localization for quantum groups at roots of unity*

Hiraku Nakajima,
*Instanton counting*

Alexander Premet,
*Minimal nilpotent representations, quantizations of Slodowy slices, and the Joseph ideal*

Eric Vasserot,
*Representations of double affine Hecke algebras*

Jie Xiao,
*Representations of tame quivers and affine canonical bases*

Contributed Talks

Susumu Ariki,
*Representation type of Hecke algebras and the Poincare polynomial*

Christopher P. Bendel,
*Cohomology for Frobenius kernels*

Brian D. Boe,
*Varieties of nilpotent matrixes for simple Lie algebras: Restricted nullcones and support varieties*

Jon F. Carlson,
*Endotrivial modules for finite groups of Lie type*

Joseph Chuang,
*Derived equivalence between blocks of GL(n)*

Stephen Doty,
*Generators and relations for generalized q- Schur algebras*
LIST OF TALKS

David J. Hemmer,
*Fixed point functors for symmetric groups and Schur algebras*

Terrell L. Hodge,
*Nilpotent orbits in restricted symmetric spaces*

James E. Humphreys,
*Representations of reduced enveloping algebras and cells in the affine Weyl group*

Dijana Jakelic,
*Crystal and tensor products in category $\mathcal{O}$*

Joel Kamnitzer,
*Mirković-Vilonen cycles and polytopes*

Masaharu Kaneda,
*Localization of $D$-modules in positive characteristic*

Gizem Karaali,
*How to construct an $r$-matrix on a Lie superalgebra*

Sergei Krutelevich,
*Exceptional groups, Jordan algebras, and higher composition laws*

Jonathan Kujawa,
*Crystal structures arising from representations of $GL(m|n)$*

Yiqiang Li,
*Affine quivers of type $\tilde{A}_n$ and canonical bases*

George J. McNinch,
*Optimal $SL(2)$-homomorphisms*

Kailash C. Misra,
*Affine Lie algebra representations and multisum identities of Rogers-Ramanujan type*

Toshiki Nakashima,
*Geometric crystals and crystal bases*

Alison Parker,
*Higher extensions for $SL_2(k)$*

Aaron Phillips,
*On 2-modular representations of the symmetric groups*

Cornelius Pillen,
*Extensions for finite groups of Lie type and the truncated induction functor*

Eric C. Rowell,
*Towards a classification of modular tensor categories*

Travis Schedler,
*Quantization of necklace Lie algebras*

Toshiyuki Tanisaki,
*The Beilinson-correspondence for quantized enveloping algebras*

Monica Vazirani,
*Vanishing integrals of Macdonald polynomials*

Weiqiang Wang,
*A super duality and Kazhdan-Lusztig polynomials*
List of Participants

Henning H. Andersen,  
*Aarhus University, DENMARK*

Susumu Ariki,  
*Kyoto University, JAPAN*

Christopher P. Bendel,  
*University of Wisconsin-Stout, USA*

Georgia Benkart,  
*University of Wisconsin Madison, USA*

Matthew Beswick,  
*Kansas State University, USA*

Brian D. Boe,  
*University of Georgia, USA*

Jonathan Brundan,  
*University of Oregon, USA*

Jon F. Carlson,  
*University of Georgia, USA*

Joseph Chuang,  
*University of Bristol, UNITED KINGDOM*

Wesley Cramer,  
*University of Virginia, USA*

Stephen Doty,  
*Loyola University Chicago, USA*

Jie Du,  
*University of New South Wales, AUSTRALIA*

Eric M. Friedlander,  
*Northern University, USA*

Fredrick Goodman,  
*University of Iowa, USA*

Holly Hauschild,  
*University of Iowa, USA*

Xuhua He,  
*Massachusetts Institute of Technology, USA*

David J. Hemmer,  
*University of Toledo, USA*

Anthony Henderson,  
*University of Sydney, AUSTRALIA*

Terrell L. Hodge,  
*Western Michigan University, USA*

James E. Humphreys,  
*University of Massachusetts-Amherst, USA*

Dijana Jakelic,  
*University of California-Riverside, USA*

Jens C. Jantzen,  
*Aarhus University, DENMARK*

Joel Kamnitzer,  
*University of California-Berkeley, USA*

Masaharu Kaneda,  
*Osaka City University, JAPAN*

Seok-Jin Kang,  
*Korea Institute for Advanced Study, SOUTH KOREA*

Gizem Karaali,  
*University of California-Berkeley, USA*

Jeon-Ah Kim,  
*Korea Institute of Advanced Study, SOUTH KOREA*

Alexander Kleshchev,  
*University of Oregon, USA*
<table>
<thead>
<tr>
<th>Name</th>
<th>Affiliation</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sergei Krutelevich</td>
<td>University of Ottawa</td>
<td>Canada</td>
</tr>
<tr>
<td>Jonathan Kujawa</td>
<td>University of Georgia</td>
<td>USA</td>
</tr>
<tr>
<td>Weiping Li</td>
<td>Walsh University</td>
<td>USA</td>
</tr>
<tr>
<td>Yiqiang Li</td>
<td>Kansas State University</td>
<td>USA</td>
</tr>
<tr>
<td>Zongzhu Lin</td>
<td>Kansas State University</td>
<td>USA</td>
</tr>
<tr>
<td>Jill E. McCarthy</td>
<td>University of Virginia</td>
<td>USA</td>
</tr>
<tr>
<td>Kevin McGerty</td>
<td>Institute for Advanced Study</td>
<td>USA</td>
</tr>
<tr>
<td>George J. McNinch</td>
<td>Tufts University</td>
<td>USA</td>
</tr>
<tr>
<td>Ivan Mirković</td>
<td>University of Massachusetts-Amherst</td>
<td>USA</td>
</tr>
<tr>
<td>Kailash C. Misra</td>
<td>North Carolina State University</td>
<td>USA</td>
</tr>
<tr>
<td>Hiraku Nakajima</td>
<td>Kyoto University</td>
<td>Japan</td>
</tr>
<tr>
<td>Daniel K. Nakano</td>
<td>University of Georgia</td>
<td>USA</td>
</tr>
<tr>
<td>Toshiki Nakashima</td>
<td>Sophia University</td>
<td>Japan</td>
</tr>
<tr>
<td>Alison Parker</td>
<td>University of Sydney</td>
<td>Australia</td>
</tr>
<tr>
<td>Brian J. Parshall</td>
<td>University of Virginia</td>
<td>USA</td>
</tr>
<tr>
<td>Julia Pevtsova</td>
<td>University of Oregon</td>
<td>USA</td>
</tr>
<tr>
<td>Aaron M. Phillips</td>
<td>University of Virginia</td>
<td>USA</td>
</tr>
<tr>
<td>Cornelius Pillen</td>
<td>University of South Alabama</td>
<td>USA</td>
</tr>
<tr>
<td>Alexander Premet</td>
<td>University of Manchester</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>Zhenbo Qin</td>
<td>University of Missouri</td>
<td>USA</td>
</tr>
<tr>
<td>Eric Rowell</td>
<td>Indiana University</td>
<td>USA</td>
</tr>
<tr>
<td>Oliver Ruff</td>
<td>University of Oregon</td>
<td>USA</td>
</tr>
<tr>
<td>Yoshihisa Saito</td>
<td>University of Tokyo</td>
<td>Japan</td>
</tr>
<tr>
<td>Travis Schedler</td>
<td>University of Chicago</td>
<td>USA</td>
</tr>
<tr>
<td>Leonard Scott</td>
<td>University of Virginia</td>
<td>USA</td>
</tr>
<tr>
<td>Dong-Uy Shin</td>
<td>Korea Institute for Advanced Study</td>
<td>South Korea</td>
</tr>
<tr>
<td>Eric Sommers</td>
<td>University of Massachusetts-Amherst</td>
<td>USA</td>
</tr>
<tr>
<td>Anna Stokke</td>
<td>University of Winnipeg</td>
<td>Canada</td>
</tr>
<tr>
<td>Toshiyuki Tanisaki</td>
<td>Osaka City University</td>
<td>Japan</td>
</tr>
<tr>
<td>Nathaniel Thiem</td>
<td>University of Wisconsin</td>
<td>USA</td>
</tr>
<tr>
<td>Michela Varagnolo</td>
<td>University of Cergy-Pontoise</td>
<td>France</td>
</tr>
<tr>
<td>Eric Vasserot</td>
<td>University of Cergy-Pontoise</td>
<td>France</td>
</tr>
<tr>
<td>Monica Vazirani</td>
<td>University of California at Davis</td>
<td>USA</td>
</tr>
<tr>
<td>Weiqiang Wang</td>
<td>University of Virginia</td>
<td>USA</td>
</tr>
<tr>
<td>Jie Xiao</td>
<td>Tsinghua University</td>
<td>China</td>
</tr>
</tbody>
</table>
Titles in This Series

414 Deguang Han, Palle E. T. Jorgensen, and David Royal Larson, Editors, Operator theory, operator algebras, and applications, 2006

413 Georgia Benkart, Jens C. Jantzen, Zongzhu Lin, Daniel K. Nakano, and Brian J. Parshall, Editors, Representations of algebraic groups, quantum groups and Lie algebras, 2006

412 Nikolai Chernov, Yulia Karpeshina, Ian W. Knowles, Roger T. Lewis, and Rudi Weikard, Editors, Recent advances in differential equations and mathematical physics, 2006


410 Abba Gumel, Carlos Castillo-Chavez, Ronald E. Mickens, and Dominic P. Clemence, Editors, Mathematical studies on human disease dynamics: Emerging paradigms and challenges, 2006

409 Juan Luis Vázquez, Xavier Cabré, and José Antonio Carrillo, Editors, Recent trends in partial differential equations, 2006

408 Habib Ammari and Hyeonbae Kang, Editors, Inverse problems, multi-scale analysis and effective medium theory, 2006

407 Alejandro Ádem, Jesús González, and Guillermo Pastor, Editors, Recent developments in algebraic topology, 2006

406 José A. de la Peña and Raymundo Bautista, Editors, Trends in representation theory of algebras and related topics, 2006

405 Andrew Markoe and Eric Todd Quinto, Editors, Integral geometry and tomography, 2006


403 Tyler J. Jarvis, Takashi Kimura, and Arkady Vaintrob, Editors, Gromov-Witten theory of spin curves and orbifolds, 2006


401 Katrin Becker, Melanie Becker, Aaron Bertram, Paul S. Green, and Benjamin McKay, Editors, Snowbird lectures on string geometry, 2006

400 Shiferaw Berhanu, Hua Chen, Jorge Hounie, Xiaojun Huang, Sheng-Li Tan, and Stephen S.-T. Yau, Editors, Recent progress on some problems in several complex variables and partial differential equations, 2006

399 Dominique Arlettaz and Kathryn Hess, Editors, An Alpine anthology of homotopy theory, 2006

398 Jay Jorgenson and Lynne Walling, Editors, The ubiquitous heat kernel, 2006

397 José M. Muñoz Porras, Sorin Popescu, and Rubí E. Rodríguez, Editors, The geometry of Riemann surfaces and Abelian varieties, 2006

396 Robert L. Devaney and Linda Keen, Editors, Complex dynamics: Twenty-five years after the appearance of the Mandelbrot set, 2006

395 Gary R. Jensen and Steven G. Krantz, Editors, 150 Years of Mathematics at Washington University in St. Louis, 2006

394 Rostislav Grigorchuk, Michael Mihalik, Mark Sapir, and Zoran Šunič, Editors, Topological and asymptotic aspects of group theory, 2006

393 Alec L. Matheson, Michael I. Stessin, and Richard M. Timoney, Editors, Recent advances in operator-related function theory, 2006

392 Stephen Berman, Brian Parshall, Leonard Scott, and Weiqiang Wang, Editors, Infinite-dimensional aspects of representation theory and applications, 2005
<table>
<thead>
<tr>
<th>Title</th>
<th>Editors</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noncommutative geometry and representation theory in mathematical physics, 2005</td>
<td>Jürgen Fuchs, Jouko Mickelsson, Grigori Rozenblioum, Alexander Stolin,</td>
<td>2005</td>
</tr>
<tr>
<td>and Anders Westerberg, Editors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commutative algebra and algebraic geometry, 2005</td>
<td>Sudhir Ghorpade, Hema Srinivasan, and Jugal Verma, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Geometry and dynamics, 2005</td>
<td>James Eells, Etienne Ghys, Mikhail Lyubich, Jacob Palis, and José Seade,</td>
<td>2005</td>
</tr>
<tr>
<td>Editors</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry, spectral theory, groups, and dynamics, 2005</td>
<td>Ravi Vakil, Editor</td>
<td>2005</td>
</tr>
<tr>
<td>Recent progress in arithmetic and algebraic geometry, 2005</td>
<td>Yasuyuki Kachi, S. B. Mulay, and Pavlos Tzermias, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Algebraic and topological dynamics, 2005</td>
<td>Sergiy Kolyada, Yuri Manin, and Thomas Ward, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Recent advances in adaptive computation, 2005</td>
<td>Z.-C. Shi, Z. Chen, T. Tang, and D. Yu, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Complex analysis and dynamical systems II, 2005</td>
<td>Mark Agranovsky, Lavi Karp, and David Shoikhet, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Coding theory and quantum computing, 2005</td>
<td>David Evans, Jeffrey J. Holt, Chris Jones, Karen Klintworth, Brian Parshall, Olivier Pfister, and Harold N. Ward, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Logic and its applications, 2005</td>
<td>Andreas Blas and Yi Zhang, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Mathematical studies in nonlinear wave propagation, 2005</td>
<td>Dominic P. Clemence and Guoqing Tang, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Groups, languages, algorithms, 2005</td>
<td>Alexandre V. Borovik, Editor</td>
<td>2005</td>
</tr>
<tr>
<td>Idempotent mathematics and mathematical physics, 2005</td>
<td>G. L. Litvinov and V. P. Maslov, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Algebraic structures and their representations, 2005</td>
<td>José A. de la Peña, Ernesto Vallejo, and Natig Atakishiyev, Editors,</td>
<td>2005</td>
</tr>
<tr>
<td>Variance and duality for cousin complexes on formal schemes, 2005</td>
<td>Joseph Lipman, Suresh Nayak, and Pramathanath Sastry, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Integer points in polyhedra—geometry, number theory, algebra, optimization, 2005</td>
<td>Alexander Barvinok, Matthias Beck, Christian Haase, Bruce Reznick, and Volkmar Welker, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Analyzable functions and applications, 2005</td>
<td>O. Costin, M. D. Kruskal, and A. Macintyre, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Geometric methods in group theory, 2005</td>
<td>José Burillo, Sean Cleary, Murray Elder, Jennifer Taback, and Enric Ventura, Editors</td>
<td>2005</td>
</tr>
<tr>
<td>Nonlinear partial differential equations and related analysis, 2005</td>
<td>Gui-Qiang Chen, George Gasper, and Joseph Jerome, Editors,</td>
<td>2005</td>
</tr>
<tr>
<td>The p-harmonic equation and recent advances in analysis, 2005</td>
<td>Pietro Poggi-Corradini, Editor</td>
<td>2005</td>
</tr>
<tr>
<td>Affine algebraic geometry, 2005</td>
<td>Jaime Gutierrez, Vladimir Shpilrain, and Jie-Tai Yu, Editors</td>
<td>2005</td>
</tr>
</tbody>
</table>

For a complete list of titles in this series, visit the AMS Bookstore at [www.ams.org/bookstore/](http://www.ams.org/bookstore/).
This book contains several well-written, accessible survey papers in many interrelated areas of current research. These areas cover various aspects of the representation theory of Lie algebras, finite groups of Lie types, Hecke algebras, and Lie superalgebras. Geometric methods have been instrumental in representation theory, and these proceedings include surveys on geometric as well as combinatorial constructions of the crystal basis for representations of quantum groups. Humphreys' paper outlines intricate connections among irreducible representations of certain blocks of reduced enveloping algebras of semi-simple Lie algebras in positive characteristic, left cells in two-sided cells of affine Weyl groups, and the geometry of the nilpotent orbits. All of these papers provide the reader with a broad picture of the interaction of many different research areas and should be helpful to those who want to have a glimpse of current research involving representation theory.