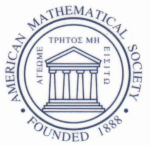
CONTEMPORARY MATHEMATICS

415

Quantum Graphs and Their Applications

Proceedings of an AMS-IMS-SIAM
Joint Summer Research Conference
on Quantum Graphs and Their Applications
June 19–23, 2005
Snowbird, Utah

Gregory Berkolaiko
Robert Carlson
Stephen A. Fulling
Peter Kuchment
Editors



American Mathematical Society

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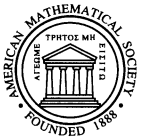

American Mathematical Society
Providence, Rhode Island

The 2006 AMS-IMS-SIAM Joint Summer Research Conference on "Quantum Graphs and Their Applications" was held at Snowbird, Utah, June 19–23, 2005, with support from the National Science Foundation, grant DMS 9973450.

2000 *Mathematics Subject Classification.* Primary 05Cxx, 11Mxx, 15A52, 28A80, 34B45, 35J05, 35Pxx, 35Q40, 58Jxx, 60J60, 81Qxx, 81U15, 92C35, 94C99.

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Preface

This volume contains contributions from the participants of the Joint Summer Research Conference "Quantum Graphs and Their Applications" that took place June 19-23, 2005 at Snowbird, Utah.

Quantum graphs are graphs considered as one-dimensional varieties and equipped with differential rather than difference operators. Such objects have appeared in recent decades in several areas of mathematics and the sciences, including chemistry, nanotechnology, microelectronics, quantum chaos, optics, number theory and dynamical systems. Prominent among the various motivations for studying such objects is that they model wave or electron propagation in thin graph-like structures. Quantum graphs is clearly a multi-disciplinary, rapidly developing subject. The goal of the conference was to promote development in the field by bringing together researchers from different areas of science and mathematics who could illuminate the subject from a variety of perspectives. The result was a stimulating and fruitful cross-pollination of ideas and methods.

The week-long conference attracted 47 participants from nine countries. They represented diverse fields, including physics, mathematical physics, algebra, combinatorics and number theory. Here is the list of lectures delivered at the conference:

- Michael Aizenman, Princeton Univ., USA. "Stability of the absolutely continuous spectra of metric tree graphs under random perturbations"
- Sergei Avdonin, Univ. of Alaska, Fairbanks, USA. "Control and Inverse Problems for Partial Differential Equations on Graphs"
- Matthew Baker, Georgia Institute of Technology, USA. "Questions about quantum graphs motivated by number theory"
- Gregory Berkolaiko, Texas A&M Univ., USA. "Correlations within the spectrum of a large quantum graph"
- Malcolm Brown, Cardiff Univ., UK. "A Borg-Levinson theorem for trees"
- Robert Carlson, Univ. Colorado, Colorado Springs, USA. "Linear network models related to blood flow"
- Fan Chung Graham, UC San Diego, USA. "Laplacians and the Cheeger inequality for directed graphs"
- Pavel Exner, Nuclear Physics Inst. and Doppler Inst., Acad. Sci., Prague, Czech Republic. "Approximations for and by quantum graph Hamiltonians"
- Mark Freidlin, Univ. of Maryland, USA. "Incompressible almost planar $3D$-flows and stochastic processes on an open book"
- Lenny Friedlander, Univ. of Arizona, USA. "The determinant of the Laplacian on a metric graph"

- Stephen Fulling, Texas A&M Univ., USA. "Local spectral density and quantum vacuum energy near a quantum graph vertex"
- Alexander Gamburd, Stanford Univ., USA. "Expander graphs, random matrices, and quantum chaos"
- Jonathan Keating, Univ. of Bristol, UK. "Eigenvalue and eigenfunction statistics for quantum star graphs"
- Jun Kigami, Kyoto Univ., Japan. "Resistance forms and *RC*-networks"
- Vadim Kostrykin, Fraunhofer Inst., Aachen, Germany. "Inverse spectral problems for quantum graphs"
- Peter Kuchment, Texas A&M Univ., USA. "On some spectral properties of combinatorial and quantum graphs"
- Pavel Kurasov, Univ. of Lund, Sweden. "Trace formulas for quantum graphs and the inverse spectral problem"
- Stanislav Molchanov, Univ. of N. Carolina, Charlotte, USA. "Localization on quantum graphs"
- Petri Ola, Nevanlinna Institute, Helsinki, Finland. "Pseudodifferential operators on graph"
- Boris Pavlov, Univ. of Auckland, New Zealand. "Quantitatively-consistent modelling of quantum networks by quantum graphs with resonance nodes"
- Koby Rubinstein, Indiana Univ., USA. "Canonical Ginzburg-Landau limits on graphs"
- Holger Schanz, Max-Planck-Inst., Göttingen, Germany. "Quantum correlations: Hamiltonian chaos vs simple graphs"
- Robert Schrader, Freie Univ., Berlin, Germany. "Quantum scattering on graphs and a solution to the Traveling Salesman Problem"
- Uzy Smilansky, Weizmann Inst., Israel. "Isospectral quantum graphs"
- Alexander Sobolev, Univ. of Sussex, UK. "Spectral statistics for the perturbed Laplace operator on the torus"
- Michail Solomyak, Weizmann Inst., Israel. "On the spectrum of a self-adjoint operator appearing in Smilansky's model of irreversible quantum graphs"
- Alexander Teplyaev, Univ. of Connecticut, USA. "Energy and spectrum of Sierpinski gasket type fractal"
- Audrey A. Terras, UC San Diego, USA. "What are zeta functions of graphs and what are they good for?"
- Boris Vainberg, Univ. of North Carolina, Charlotte, USA. "Slowing down wave packets in quantum graphs"

Besides well established researchers, there were 13 undergraduate and graduate students and postdocs among the attendees. They were able to represent their results at the poster sessions.

The participants are looking forward to a related semester long program, "Analysis on Graphs and its Applications," at the Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, in January – June 2007.

Besides the very high scientific level of the participants and presentations, the wonderful scenery of the ski resort and the superb organization by the AMS staff have contributed to the success of the conference.

The editors (who also organized the conference) express their gratitude to all the participants and contributors to this volume, to AMS staff members Wayne

Drady and Donna Salter for their flawless work on the organization of the conference and to Christine Thivierge and other members of the AMS technical staff for great help with editing this volume.

Gregory Berkolaiko, Texas A&M University
Stephen A. Fulling, Texas A&M University
Robert Carlson, University of Colorado, Colorado Springs
Peter Kuchment, Texas A&M University

Titles in This Series

415 **Gregory Berkolaiko, Robert Carlson, Stephen A. Fulling, and Peter Kuchment, Editors,** Quantum Graphs and Their Applications, 2006

414 **Deguang Han, Palle E. T. Jorgensen, and David Royal Larson, Editors,** Operator theory, operator algebras, and applications, 2006

413 **Georgia Benkart, Jens C. Jantzen, Zongzhu Lin, Daniel K. Nakano, and Brian J. Parshall, Editors,** Representations of algebraic groups, quantum groups and Lie algebras, 2006

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This volume is a collection of articles dedicated to quantum graphs, a newly emerging interdisciplinary field related to various areas of mathematics and physics. The reader can find a broad overview of the theory of quantum graphs. The articles present methods coming from different areas of mathematics: number theory, combinatorics, mathematical physics, differential equations, spectral theory, global analysis, and theory of fractals. They also address various important applications, such as Anderson localization, electrical networks, quantum chaos, mesoscopic physics, superconductivity, optics, and biological modeling.

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