

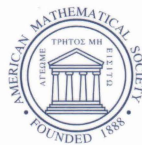
# CONTEMPORARY MATHEMATICS

424

## The Interaction of Analysis and Geometry

International School-Conference  
Analysis and Geometry  
August 23–September 3, 2004  
Novosibirsk, Russia

V. I. Burenkov  
T. Iwaniec  
S. K. Vodopyanov  
Editors



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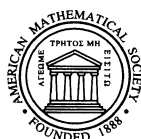
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## Preface

The volume contains the papers of participants of the International Conference on Analysis and Geometry in honor of the 75th birthday of Yuriĭ Reshetnyak, an outstanding mathematician, a member of the Russian Academy of Science. His prominent investigations of two-dimensional manifolds of bounded curvature and mappings with bounded distortion are an outstanding contribution into the mathematics. The conference was held through August 23 – September 3, 2004 in Novosibirsk.

The scope of the volume includes geometry of spaces with bounded curvature in the sense of Alexandrov, quasiconformal mappings and mappings with bounded distortion (quasiregular mappings), nonlinear potential theory, Sobolev spaces, spaces with fractional and generalized smoothness, and variational problems.

The volume reflects modern trends in these areas. Most articles are related to Reshetnyak's original works and demonstrate the vitality of his fundamental contribution in some important fields of mathematics such as the geometry in the "large", quasiconformal analysis, Sobolev spaces, potential theory and variational calculus.

The topics discussed in the volume can be subject of research for many mathematicians. We hope that this volume will be a valuable source both for experts and young researchers by providing them with a wealth of information, which otherwise is scattered in literature or not published at all.

V. Burenkov, T. Iwaniec and S. Vodopyanov  
September, 2006

The papers in this volume are based on talks given at the International Conference on Analysis and Geometry in honor of the 75th birthday of Yuriĭ Reshetnyak (Novosibirsk, 2004). The topics include geometry of spaces with bounded curvature in the sense of Alexandrov, quasiconformal mappings and mappings with bounded distortion (quasiregular mappings), nonlinear potential theory, Sobolev spaces, spaces with fractional and generalized smoothness, variational problems, and other modern trends in these areas. Most articles are related to Reshetnyak's original works and demonstrate the vitality of his fundamental contribution in some important fields of mathematics such as the geometry in the "large", quasiconformal analysis, Sobolev spaces, potential theory and variational calculus.

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