Stochastic Analysis and Partial Differential Equations

Northwestern University
Evanston, Illinois

Gui-Qiang Chen
Elton Hsu
Mark Pinsky
Editors
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American Mathematical Society
Providence, Rhode Island
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PREFACE

This book is a collection of refereed original research papers and expository articles from the scientific program of the 2004–05 Emphasis Year on Stochastic Analysis and Partial Differential Equations at Northwestern University. The Emphasis Year was jointly sponsored by Northwestern University and the National Science Foundation. It was an occasion for a yearlong series of seminars, invited lectures and courses, and a research conference. The International Conference on Stochastic Analysis and Partial Differential Equations was held during June 26–30, 2005, for which there were approximately 100 participants.

This volume comprises sixteen papers, mostly contributed by the invited speakers in the conference and seminars. Contributions range from stochastic analysis of turbulence, Markov processes, microscopic lattice dynamics, microscopic interacting particle systems, and stochastic analysis on manifolds to partial differential equations including kinetic equations, hyperbolic conservation laws, Navier-Stokes equations, and Hamilton-Jacobi equations. A variety of methods, including numerical analysis, homogenization, measure-theoretical analysis, entropy analysis, weak convergence analysis, Fourier analysis, and Itô's calculus, are further developed and applied. These topics are naturally interrelated and represent a cross-section of the most significant recent advances and current trends and directions in stochastic analysis and partial differential equations.

The editors would like to express their gratitude to all the participants in the Emphasis Year, to the authors of the papers in this volume, and to the referees for their constructive comments and suggestions. The editors are grateful to the American Mathematical Society, especially, Dennis DeTurck (Managing Editor of Contemporary Mathematics), Edward G. Dunne (Editor of the Book Program), and Christine M. Thivierge (Acquisitions Assistant) for their patience and their prompt and professional assistance.

The tradition of the Northwestern University Emphasis Year Program dates back to the academic year 1973–74. In particular, the first of the successful series of Midwest Partial Differential Equations Seminars was held in 1977 and the First International Conference on Stochastic Analysis was held in 1978 at Northwestern University. The editors express their thanks to the National Science Foundation and Northwestern University, more specially, the Department of Mathematics and the Weinberg College of Arts and Sciences for their continuous support of this excellent legacy.
Finally, the editors wish to acknowledge the financial support of the National Science Foundation through the NSF-Grant DMS-0426172 for the Emphasis Year 2004–05.

Gui-Qiang Chen  Elton P. Hsu  Mark Pinsky

November 18, 2006
Evanston, Illinois, USA
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This book is a collection of original research papers and expository articles from the scientific program of the 2004-05 Emphasis Year on Stochastic Analysis and Partial Differential Equations at Northwestern University. Many well-known mathematicians attended the events and submitted their contributions for this volume.

Topics from stochastic analysis discussed in this volume include stochastic analysis of turbulence, Markov processes, microscopic lattice dynamics, microscopic interacting particle systems, and stochastic analysis on manifolds. Topics from partial differential equations include kinetic equations, hyperbolic conservation laws, Navier–Stokes equations, and Hamilton–Jacobi equations. A variety of methods, such as numerical analysis, homogenization, measure-theoretical analysis, entropy analysis, weak convergence analysis, Fourier analysis, and Itô's calculus, are further developed and applied. All these topics are naturally interrelated and represent a cross-section of the most significant recent advances and current trends and directions in stochastic analysis and partial differential equations.

This volume is suitable for researchers and graduate students interested in stochastic analysis, partial differential equations, and related analysis and applications.