# CONTEMPORARY MATHEMATICS

444

## Topics in Harmonic Analysis and Ergodic Theory

December 2–4, 2005 DePaul University, Chicago, Illinois

> Joseph M. Rosenblatt Alexander M. Stokolos Ahmed I. Zayed Editors



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2000 Mathematics Subject Classification. Primary 37A30, 37A50, 42A63, 42B30, 42B35, 42C15, 42C40, 47A35, 60F05, 65D10.

### Library of Congress Cataloging-in-Publication Data

 $Topics \ in \ harmonic \ analysis \ and \ ergodic \ theory \ / \ Joseph \ M. \ Rosenblatt, \ Alexander \ M. \ Stokolos, \ Ahmed \ I. \ Zayed, \ editors.$ 

p. cm. — (Contemporary mathematics, ISSN 0271-4132 ; v. 444) Includes bibliographical references.

ISBN 978-0-8218-4235-5 (alk. paper)

1. Harmonic analysis—Congresses. 2. Ergodic theory—Congresses. Geometry—Data processing—Congresses. I. Rosenblatt, J. (Joseph) M., 1946– II. Stokolos, Alexander M., 1960– III. Zayed, Ahmed I.

QA403.T5657 2007 515'.2433—dc22

2007060789

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### Preface

The idea of this volume was conceived during a conference on Harmonic Analysis and Ergodic Theory that took place at DePaul University, Chicago, December 2-4, 2005. The conference was funded by grants from the National Science Foundation and the DePaul University Research Council. Originally, the conference was planned as a small meeting to honor two members of the DePaul Department of Mathematical Sciences, J. Marshall Ash and Roger L. Jones; Marshall Ash for his 65th birthday and Roger Jones because of his retirement after a long and distinguished career at DePaul. Ash and Jones are well known for their contributions in harmonic analysis and ergodic theory, respectively. But very soon the idea of a small meeting turned into a full blown meeting with 74 participants.

From the outset the meeting had to be planned in a somewhat unusual format because it had two main themes and we were not in favor of having parallel sessions. Although harmonic analysis and ergodic theory are intimately related, there are few mathematicians who are experts in both. Therefore, to make the meeting more appealing to graduate students and junior researchers, we decided to imbed two mini-courses in it, one on harmonic analysis, and another on ergodic theory. The mini-course on harmonic analysis, which was given by Professor Carlos Kenig, University of Chicago, focused on harmonic analysis techniques in solving nonlinear partial differential equations, while the one on ergodic theory, which was given by Professor Daniel Rudolph, Colorado State University, concentrated on ergodic theory structures for foliations.

In addition to the mini-courses, the meeting had plenary talks and sessions for contributed papers. The plenary speakers in alphabetical order were:

- Vitaly Bergelson, Ohio State University
- Charles Fefferman, Princeton University
- Richard Gundy, Rutgers University
- Carlos Kenig, the University of Chicago
- Michael Lacey, Georgia Institute of Technology
- Michael Lin, Ben-Gurion University of the Negev Beer-Sheva
- Fedor Nazarov, Michigan State University
- Joseph Rosenblatt, the University of Illinois at Urbana-Champaign
- Daniel Rudolph, Colorado State University
- Arkady Tempelman, Pennsylvania State University
- Stephen Wainger, the University of Wisconsin-Madison
- Mate Wierdl, the University of Memphis.

This volume does not constitute the proceedings of the conference, but rather a collection of articles by some of the plenary speakers based on their talks. Contributions to the volume were by invitation only. All articles were carefully refereed

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and present either original research papers or high quality survey articles. The chapters are almost equally divided between the two themes.

The Editors are grateful to the authors and co-authors for the time and efforts they put in writing, revising, and formatting their work according to the high standards set by the American Mathematical Society. We are also indebted to the referees for their meticulous reports. Last but not least, we would like to thank the Editorial Board of the Contemporary Mathematics Series for publishing this volume and Christine M. Thivierge, Editorial Assistant, American Mathematical Society, for her help and guidance throughout the editorial and production processes.

The Editors,

Joseph M. Rosenblatt, Alexander M. Stokolos and Ahmed I. Zayed June 2007

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Zeytuncu, Yunus Ohio State University There are strong connections between harmonic analysis and ergodic theory. A recent example of this interaction is the proof of the spectacular result by Terence Tao and Ben Green that the set of prime numbers contains arbitrarily long arithmetic progressions. The breakthrough achieved by Tao and Green is attributed to applications of techniques from ergodic theory and harmonic analysis to problems in number theory.

Articles in the present volume are based on talks delivered by plenary speakers at a conference on Harmonic Analysis and Ergodic Theory (DePaul University, Chicago, December 2–4, 2005). Of ten articles, four are devoted to ergodic theory and six to harmonic analysis, although some may fall in either category. The articles are grouped in two parts arranged by topics. Among the topics are ergodic averages, central limit theorems for random walks, Borel foliations, ergodic theory and low pass filters, data fitting using smooth surfaces, Nehari's theorem for a polydisk, uniqueness theorems for multi-dimensional trigonometric series, and Bellman and s-functions.

In addition to articles on current research topics in harmonic analysis and ergodic theory, this book contains survey articles on convergence problems in ergodic theory and uniqueness problems on multi-dimensional trigonometric series.



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