

CONTEMPORARY MATHEMATICS

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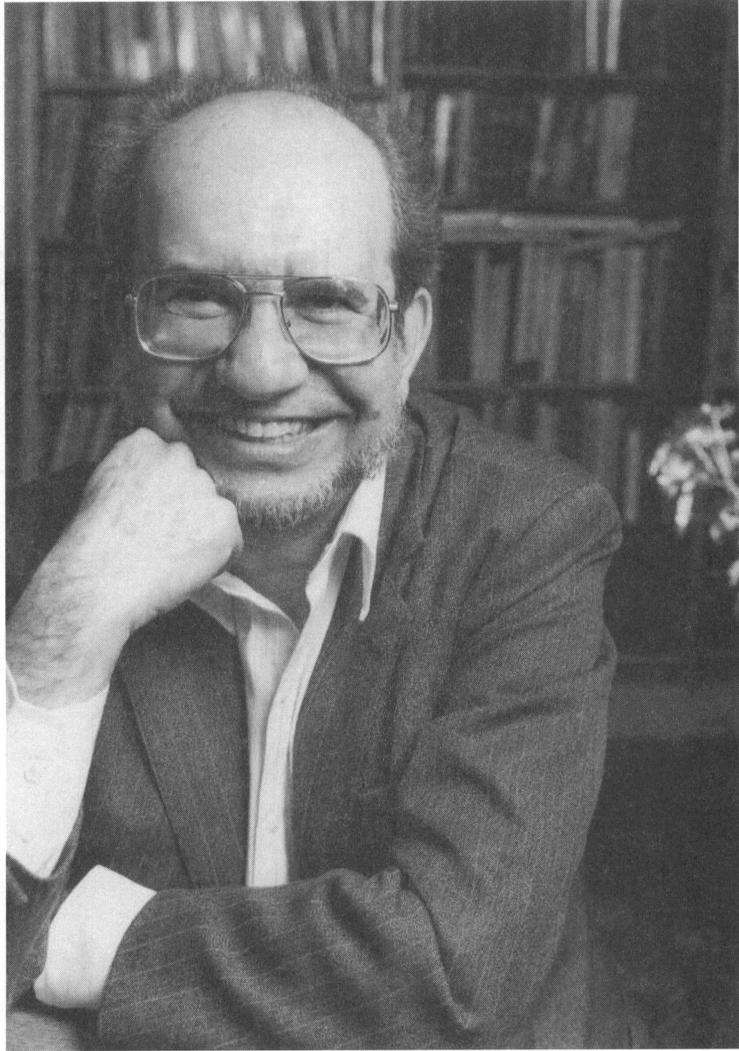
Perspectives in Nonlinear Partial Differential Equations

In Honor of Haïm Brezis

Henri Berestycki
Michiel Bertsch
Felix E. Browder
Louis Nirenberg
Lambertus A. Peletier
Laurent Véron
Editors



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Haim Brezis

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Preface

On June 1, 2004, Haïm Brezis turned 60, an occasion that was celebrated by scientific conferences held in the USA, Italy and France. The Carré des Sciences in the old building of the École Polytechnique, now the Ministère de la Recherche, provided a fitting backdrop for the Paris meeting, which was held on June 21-25, 2004. An atmosphere of personal warmth and the scientific breadth and depth made this occasion memorable. The program testified to the wide ranging influence of Haïm on the field of nonlinear analysis and partial differential equations. In particular, his unique role in stimulating research and opening new directions was emphasized. Many lecturers had the occasion to recall Haïm's special passion and talent for open problems and his continuous un-abiding discoveries of gem-like challenging questions. Many of the speakers also spoke of the generosity of Haïm in welcoming and encouraging young new researchers.

As a tribute paid to Haïm Brezis, the lecturers at the Paris meeting made a particular effort to put together a rare view of the state of the art of many aspects of nonlinear partial differential equations, as well as to describe new directions that are being opened up in this field, and some of the challenges it faces.

In the same spirit, it was felt that such perspectives should be gathered and recorded through a collection of articles that make up this volume. Written by mathematicians at the center of current developments, many of whom spoke at the Paris meeting, this volume provides somewhat more personal views of the important directions and challenges.

The American Mathematical Society responded with enthusiasm to this project and supported it with high-quality professional help. We are delighted that this volume appears in the scientifically-demanding Contemporary Mathematics Series. It is with pleasure that we offer thanks to the publishing team of the AMS, in particular to Edward Dunne and Christine Thivierge. We also wish to express our thanks to Ann Kostant of Birkäuser, Boston, who was very supportive of the initiative.

Besides the mathematical articles, two memorable speeches have been transcribed and included here, at the beginning of this volume. They were delivered by Haïm and by Michal Govrin, his wife, a Hebrew writer, at a party in honor of Haïm during the meeting in Paris. They convey something of the special atmosphere of this meeting. We are grateful to them for granting us permission to print these personal thoughts in this volume.

The Editors

On Haïm Brezis

A volume offering a perspective on the state of the art of nonlinear partial differential equations is a fitting tribute to someone whose ideas and contributions are so ubiquitous throughout the subject. All the more so, it seemed to us, when these perspectives aim at shedding light on future directions and challenges that lie ahead of the subject. Haïm Brezis is indeed known to have relentlessly scrutinized the mathematical horizon and has always shown an unquenchable passion for open problems.

To describe in any kind of detail how much the field of nonlinear PDE's owes to Haïm Brezis would be an immense task, one that goes well beyond the scope of this brief introduction. Nor is it our purpose here. We only wish to paint with a few broad brush strokes some of the landscape that Haïm's activity has profoundly changed (and sometimes created), leaving the finer detail for others to fill in. His list of publications - already incomplete! - that is included at the end of this volume is the prime source for more detailed information.

A striking feature of Haïm's many contributions is the variety of problems to which he has made seminal contributions. The spectrum of his work spans from the very abstract to specific equations arising in physics. Haïm grew up as a mathematician in the effervescent environment created by Laurent Schwartz, Gustave Choquet and Jacques-Louis Lions in Paris. He started his career as a junior researcher at the French Centre National de la Recherche Scientifique. As Jacques-Louis Lions had strong ties with Guido Stampacchia and with the Italian school in general, Haïm Brezis had the occasion early on to travel to Pisa. Under their joint influence, he became interested in problems from mechanics, involving unilateral constraints and variational inequalities. In collaboration with Guido Stampacchia and later with David Kinderlehrer, in particular, he established regularity results for variational inequalities involving the Dirichlet integral that proved to be important for deriving estimates for the regularity of free boundaries. Continuing in this direction, he developed applications of variational inequalities to specific problems in mechanics such as subsonic flow around a profile (using the hodograph method), flow of water through a porous dam and the evolution of ice-water mixtures.

Soon afterwards, Haïm achieved his first major results in the theory of maximal monotone operators and their generalizations and in the theory of semigroups of contractions. These bear the strong influence of Felix Browder. This program aimed at laying the theoretical foundations of evolution equations involving operators of monotone type. It led to a unified framework for a wide range of nonlinear

evolution equations allowing one to treat equations that do not have classical solutions. The Porous Media equation and Hamilton-Jacobi equations are important examples of applications of this theory. In 1973, this body of work resulted in the well-known monograph by H. Brezis: *Opérateurs Maximaux Monotones et semi-groupes de contractions dans les espaces de Hilbert*.

Already present in this monograph was the style of Haïm that was to become his trademark in mathematical writing: impeccable rigor, concision and sharpness allied with the elegance and the pursuit of an always more beautiful and enlightening presentation. This style is also the expression of Haïm's ethical attitude that strives to make mathematical writing as welcoming as possible, especially to young new researchers, without giving up an inch of generality or rigor.

From there, Haïm Brezis started to work on semi-linear elliptic equations in the space $L^1(\mathbb{R}^N)$. The next step was to consider such problems with measures. The techniques he had developed proved fruitful in analyzing the Thomas-Fermi equation arising in atomic physics. Haïm Brezis thus embarked on a detailed study of semi-linear elliptic equations with measures as data. Of primary concern in this context is the question of removability of singularities. Here another trademark of Haïm's research becomes apparent. A way of picking up from physics hard and challenging questions and extracting a kind of mathematical quintessence. In many instances, these problems, on which Haïm paved the way, gave rise to a considerable activity which is ongoing.

The mathematical study of the Thomas-Fermi equation was initiated by H. Brezis in 1977 with Philippe Bénilan, but the article, although well distributed in the form of an endlessly photocopied manuscript, remained unpublished until Benilan's death. As a tribute to his first student's mathematical work, Haïm undertook to write the full extent of their research in a long and beautiful paper that appeared in 2003 in the special volume dedicated to Philippe Bénilan. Further developments on these types of problems, some by Haïm's students, showed that the behavior near singularities was critically linked to sharp threshold values associated with the dimension of the underlying Euclidean space. These types of problems generated many further studies in the past thirty years. Compare, for instance, the contribution of Moshe Marcus and Laurent Véron in this volume.

The previous problems have the structure of semi-linear elliptic equations. Haïm was naturally led to study more generally these types of problems. In 1965, Stanislas Pohozaev pointed out a critical threshold value for the growth of superlinear problems with power nonlinearities. As a consequence, existence of a positive solution of the Dirichlet problem for semilinear elliptic equations in a bounded star-shaped domain with zero boundary data depends on the growth rate of the power nonlinearity in the equation. At a critical power p_c the Sobolev embedding of $L^p(\Omega)$ into $H^1(\Omega)$, where Ω is a bounded domain, ceases to be compact. In 1982, Louis Nirenberg and Haïm Brezis studied the case of the critical Sobolev growth that arises in the Yamabe problem. They introduced a linear term involving an eigenvalue parameter and were able to give a sharp description. They showed that solvability depends in a subtle way on the dimension of the underlying space and on

the geometry of the domain. This equation and variations of it revealed the richness of the structure of these problems near the threshold of compactness. Indeed, it required the development of fundamentally new ideas, since the functional that arises in the variational formulation of the Dirichlet problem no longer satisfies the Palais-Smale compactness condition.

In retrospect, this classical article from 1982 has opened the way to a vast literature that explored the implications of these phenomena with ramifications reaching into differential geometry and algebraic topology and with applications to physics. A central line in Haïm's work was initiated on this problem. It deals with various situations that are borderline from the point of view of compactness. In fact, this is a fascinating situation, rich in structure, often encountered in geometry, where it motivated earlier studies of this phenomena, and in physics. "When compactness fails" became a recurring theme in the work of Haïm Brezis that followed. This gave birth to papers with Jean-Michel Coron on classical problems arising in geometry, such as "large bubble" solutions for constant mean curvature surfaces and harmonic maps.

Haïm's interest in problems with origins in the physical sciences was also clearly present in his work on liquid crystals. It grew out of discussions that he had with Jerry Ericksen during a special year at the Institute for Mathematics and its Applications in Minneapolis. This led to another development: the minimization of functionals defined on functions with values on the unit sphere in \mathbb{R}^3 . The functions essentially represent the orientation of molecules. The nature of the singularity in a minimizing configuration was an outstanding open problem. Likewise, for a collection of singularities of given degrees, one also wanted to understand what is the infimum of the energy needed to maintain it. These two central questions were solved completely in the important case of harmonic mappings (a model for the general liquid crystal problem) in the work of Haïm with Jean-Michel Coron and Elliot Lieb.

It would be hard to overstate the importance of Haïm's work on the Ginzburg-Landau theory, which was developed in the 1950's for the study of superconductivity and super-fluids. In the absence of external forces, this leads to the minimization of an energy functional involving a complex valued *order variable* and a small length parameter ε . From the point of view of physics, an essential question here concerns the asymptotic behavior as $\varepsilon \rightarrow 0$. If the problem is defined on a bounded domain Ω in \mathbb{R}^2 and on the boundary $\partial\Omega$ the order parameter is prescribed by a certain function g with a given degree d , then if $d \neq 0$, singularities carrying infinite energy will develop in the interior of Ω as $\varepsilon \rightarrow 0$. This gives rise to the questions of how many singularities eventually will develop when ε becomes very small, what their character is and where in Ω they will be located. These problems led to an enormous mathematical effort to understand them as well as to obtain answers in more general situations, involving for instance, magnetic fields.

Thanks to the now classical monograph by Fabrice Bethuel, Haïm Brezis and Frédéric Hélein, *Ginzburg-Landau vortices*, published in 1994 by Birkhäuser, the early developments in this theory became readily accessible to a wide audience of

mathematicians and of other scientists. It has become an inspiring topic for graduate seminars. Again, Haïm's welcoming style, as well as his clarity and elegance, opened up this topic and made the techniques that the authors had developed available to a large audience, especially to new researchers.

This work enjoyed a considerable posterity both from the point of view of mathematical studies and extensions to new classes of problems arising in physics as is clear from contributions appearing in the present volume. We refer in particular to the articles by Amandine Aftalion and Sylvia Serfaty.

One of the many positive outgrowths of Haïm's works on Ginzburg-Landau equations is the theory of lifting of functions with values in the unit circle S^1 . With Jean Bourgain and Petru Mironescu, he studied the conditions that allow a function $u \in H^1(\Omega; S^1)$ to be written in the form $u = e^{i\phi}$ with $\phi \in H^1(\Omega; \mathbb{R})$. This followed another insight in degree theory accomplished with Louis Nirenberg, where the degree of a map is defined not only for a continuous function, but also for a function with vanishing mean oscillation. This extension of the notion of degree was a key step in establishing a series of results in which the classical argument of degree theory dealing with continuous functions can no longer be used. Haïm's ongoing research, among other things, continues this theme and looks in a very refined way at properties of generalized functions. This line of problems is present here in the article of Fabrice Bethuel and David Chiron and in the one of Petru Mironescu.

Looking over Haïm's work, one is awed by the many new directions he has identified and to which he has given the first major impulse: critical growth in semi-linear equations, variational analysis of functionals in borderline compactness cases, vector valued equations, the Ginzburg-Landau theory as well as generalized degree theory and fine properties of Sobolev spaces. All of these exhibit the mark of Haïm's genius.

Haïm's achievements in mathematics have been honored by the many prizes he received: the Peccot Prize (1974), the Carrière Prize (1976), the Ampère Prize (1985) of the French Academy of Sciences, the E. Catalan Prize from the Académie Royale de Belgique (1990), the Ky Fan prize of the American Mathematical Society (2001). A member of the French Académie des Sciences since 1988 and the Academia Europæa, he is also a foreign member of the Academia Romana, the Academy of Arts and Sciences and the National Academy of Sciences of the US, the Real Academia de Ciencias de España and the Académie Royale de Belgique. The recognition of his scientific achievements is also evident from a long list of honorary doctorates or professorships bestowed upon him by universities all over the world. Haïm Brezis has been a member of the Institut Universitaire de France since 1997.

However, it is not only through his mathematical ideas and results that Haïm has had an impact on the agenda of nonlinear partial differential equations. From

early on, his personal and tireless commitment to the subject and to its practitioners, young and old, has considerably influenced this field.

At Université Pierre et Marie Curie (Paris VI), at Jussieu, where he has been a professor since 1972, Haïm proved to be an inspiring teacher. His course *Analyse Fonctionnelle* became a classic with packed classrooms of students in awe at the beauty of the subject. The way Haïm turned teaching this subject into an art made this course extremely successful and attracted many young researchers to this field. The textbook that came out of this course has become a bestseller, and has been translated from French into eight languages: Chinese, English, Greek, Italian, Japanese, Korean, Romanian and Spanish. At Paris VI and at École Polytechnique, where he taught from 1973 to 1985, Haïm has formed generations of French mathematicians and, even for the many students who did not pursue mathematical careers, his courses remain examples of outstanding presentations.

Many students however were naturally drawn to mathematics through him and more than fifty of them have completed their PhD thesis under his guidance. In turn, many of his students have become well-established mathematicians contributing greatly to the subject and to the mathematical community. It is widely acknowledged that in this too, Haïm exhibits an extraordinary talent. He always finds a welcoming problem for a beginner, one that is not discouraging and yet is a good entrance into the subject. This, of course, is related to the special passion and art Haïm has for open problems, the special attention he always paid to the elegance of proofs. It was not enough that the problem be solved; it had to be done properly, with a demanding sense of esthetics. Arguments had to enlighten a question. The progress of the students, from the first attempts up to the complete solution, were all greeted by the warm and gratifying enthusiasm of Haïm. Anecdotes about the years of apprenticeship with Haïm are remembered with affection by his former students.

Haïm's scientific influence went beyond his students. In the seventies, it was already felt through his eminent role in the prestigious Brezis-Lions seminar on *Nonlinear Partial Differential Equations and their Applications*, which was held on Friday afternoons at the Collège de France from 1977 until the early 1990's. A meeting place for everyone who was interested in nonlinear PDE's in Paris and beyond, as some people travelled from other cities in Europe to attend it. The penetrating questions of Haïm and Jacques-Louis Lions often opened up new problems or new perspectives on the subject of the day's seminar.

Certainly, the talent of Haïm as a lecturer made him well known to large audiences well beyond the field of nonlinear PDE's. With his characteristic energy and enthusiasm and an extraordinary art for mathematical narrative, Haïm captivates his audiences almost as in a suspense film with a masterly screenplay. He is able to carry with him whole audiences to the most subtle and technically involved levels. In Haïm's lectures, the high sense of aesthetics is combined with an ethical urge to explain and reveal the mathematical truth, one that should be made available to everyone.

In 1988, Haïm was elected to the Académie des Sciences in Paris. He became a very active member of this prestigious institution, in particular in numerous committees and in editing the *Comptes-Rendus de l'Académie des Sciences*. This is but one instance of his intense editorial activity. Haïm has worked with great commitment on the publishing of mathematical research. As the main editor of the series *Progress in Nonlinear Partial Differential Equations* at Birkäuser, he encouraged young researchers to present their research in monographs. A tireless editor, on the board of 29 mathematical journals, he has been, and is committed to, the promotion of scientific quality in publication. In particular, for a few years now he has been a very active chief editor of the *Journal of the European Mathematical Society (JEMS)*, again striving for excellence.

International cooperation among mathematicians has always been a high priority for Haïm. He has felt strongly about this almost from the start of his mathematical life. He had the occasion to travel to Pisa, to Israel and to the Courant Institute in New York, and then to the University of Chicago. These experiences made a profound impression on him. Early on, he took several students from Spain who turned out to become important mathematicians inspiring a whole generation, thanks to whom the field of nonlinear analysis flourished in Spain. Students have been attracted to Haïm from all over the world: the Netherlands, Israel, South America, North Africa, China, Eastern Europe... Likewise, he became a mentor for postdoctoral students who came from all over the world to Paris or to Rutgers in order to work with him. An avid and enthusiastic traveller, Haïm has shown a great curiosity for cultures around the world. He dedicated much effort and energy to promote international cooperation by forging institutional relationships with Paris VI on bilateral bases and then by taking a prominent part in European networks. He thus gave critical support to groups active in nonlinear partial differential equations in countries such as Spain, the Netherlands and Romania, to name but a few. With positions at Paris VI, Rutgers and Technion, as chief editor of the *Journal of the European Mathematical Society*, and as Vice-President of the *American Mathematical Society*, Haïm's influence has spread to many places. Over the years, it has become evident that Haïm is a true *global mathematician*, one for whom geographical boundaries count little and who, in a way, is a citizen of the mathematical world.

All these mathematical elements, along with his many friends, were in evidence at the celebration of Haïm's 60th birthday on a few sunny days at the Carré des Sciences, in the former buildings of the *École Polytechnique* in the Center of Paris, in June 2004. It was a unique occasion filled with warmth and gratitude.

Alongside these various facets of this professional career, one ought to evoke Haïm's personality; an extremely rich one indeed. Having strong convictions, Haïm is also a warm and attendant person, one who pays much attention to others. Also, it would be hard not to mention his strong ties to Judaism and to Israel. Jewish study occupies a central place in his life. This is not the place to elaborate on these themes. In his book¹, Haïm has expressed in detail and with eloquence his views on these subjects and how they intertwine with his life as a mathematician.

¹Haïm Brezis un mathématicien juif, *entretiens avec Jacques Vauthier*, Beauchesne, Paris, 1999

One can also get a glimpse into these more personal aspects from the speeches that Haïm and his wife, the Israeli writer Michal Govrin, gave at the party in his honour at Université Paris VI on the previous mentioned occasion of the international meeting. Both of these speeches are reproduced in this volume.

Henri Berestycki

Michiel Bertsch

Felix Browder

Louis Nirenberg

Lambertus A. Peletier

Laurent Véron

Allocution de Haïm Brezis le 22 Juin 2004 à l'Université Pierre et Marie Curie lors de la réception en l'honneur de son 60eme anniversaire

Monsieur le Président de l'Université,
Monsieur le Président de l'Académie des Sciences,
Madame la Maire,
Monsieur le Directeur Général du CNRS,
Chers amis et collègues,

Dans un article intitulé *Les processus mentaux de la création*, Gustave Choquet, qui fut l'un de mes premiers maîtres, écrit: "A trente ans on cherche et on trouve. Un peu plus tard [vers 40 ans, je suppose] on forme des chercheurs, et plus tard encore [vers 50 ans ?] on parle des processus mentaux de la création". Gustave Choquet ne nous dit rien pour la suite. Est-on réduit à écrire des lettres de recommandation? à faire des discours pour les anniversaires de ses collègues? ou peut-être simplement à garder le silence?

Bienvenue à vous tous dans cette maison qui est ma maison depuis 40 ans. Si je devais résumer en deux mots mon sentiment vis-à-vis de cette vénérable institution, je dirais qu'elle représente pour moi un extraordinaire espace de liberté et d'échanges. Échanges et liberté.

Liberté. Tout d'abord au sens de tolérance, d'ouverture vers l'autre. En 1964, j'arrivais dans cette maison, complètement désorienté et même un peu déprimé. En effet, l'École Normale Supérieure de la rue d'Ulm avait claqué la porte au nez du citoyen Brezis qui s'entêtait à respecter ses convictions religieuses et à ne pas vouloir écrire le samedi. A défaut d'École Normale, je me suis "consolé" en arrivant ici, perplexe quant à mon avenir mathématique en France.

Dès la première année ce fut le coup de foudre. J'ai été littéralement ébloui par des maîtres tels Claude Chevalley, François Bruhat et Roger Godement; ensuite par Gustave Choquet et Laurent Schwartz. Et plus tard, par Jacques-Louis Lions qui a eu un impact considérable sur l'orientation de mes recherches. Je voudrais affirmer haut et fort que, depuis mon entrée à Paris VI, je n'ai jamais rencontré d'entrave due à mes origines ou à mes convictions. En d'autres temps, c'eût été une évidence. Dans le climat actuel, cela mérite d'être souligné.

Autre dimension de la liberté: liberté totale dans la créativité. J'ai toujours choisi de travailler sur des sujets de recherche qui me passionnaient. Personne – je dis bien personne – ne m'a imposé une direction de recherche. J'ai aussi eu le privilège de donner des cours de Maîtrise et de 3eme cycle sur des sujets qui me

captivaient et même le loisir de façonner des programmes à mon goût. Et grâce à cette liberté, j'ai pu transmettre mon propre enthousiasme aux jeunes étudiants.

Bien entendu, cette liberté s'est encore accrue depuis que je suis devenu membre de l'Institut Universitaire de France – une institution extraordinaire. Et je ne manque pas une occasion de dire toute ma reconnaissance à Yves Meyer, un ami très cher, pour son soutien à un moment crucial de ma carrière.

Liberté de mouvement aussi. Mes collègues et mes élèves ont été remarquablement tolérants vis-à-vis de ma mobilité – et je leur en suis très reconnaissant. Ils ont compris que si je pars souvent en voyage, c'est pour faire du tourisme d'un type spécial – le tourisme mathématique – et pour ramener du monde entier des idées nouvelles glanées ici et là. Très jeune, je me suis rendu dans trois centres mathématiques qui ont eu une influence importante sur ma formation.

Tout d'abord Pise, à l'âge de 23 ans, où j'ai eu le privilège de rencontrer Guido Stampacchia et Ennio DeGiorgi. Ils sont malheureusement disparus, mais je pense souvent à eux et suis touché que l'école mathématique italienne soit si bien représentée ce soir.

Ensuite, je suis parti pour un long séjour au Courant Institute, à New York, où j'ai eu la chance de recevoir la flamme mathématique des mains de Louis Nirenberg. Je le remercie très vivement pour tout ce que j'ai appris de lui et pour sa présence parmi nous.

Enfin, je me suis souvent rendu, dans les années 70, à l'Université de Chicago sur l'invitation de Felix Browder. Grâce à Felix j'ai pris goût à la très belle théorie des opérateurs monotones. Nous avons ensuite beaucoup collaboré et, au fil des années, Felix est devenu un proche ami. Felix souhaitait vivement participer à ce Colloque; malheureusement, pour des raisons de santé, il a dû renoncer à ce projet. Mais les fruits de ses efforts sont manifestes puisqu'il s'est chargé d'obtenir une subvention financière de la National Science Foundation pour ce Colloque. A ce propos, je voudrais saluer la présence parmi nous de Hans Kaper, Directeur de la division des mathématiques appliquées à la NSF. Je note avec joie et fierté que la crème des Universités américaines sont représentées ici: l'Institute for Advanced Study de Princeton, les Universités de Princeton, de Chicago, Rutgers, Madison, NYU, Berkeley, Austin, Carnegie-Mellon et d'autres encore.

Ceci m'amène à mon second thème: notre Université comme espace d'échange dans toutes ses dimensions. Échanges entre juniors et seniors. Échanges entre français et étrangers. Échanges entre pays à la pointe du progrès scientifique et pays en voie de développement. J'ai été ravi d'accueillir tout au long de ma carrière des jeunes venus de Roumanie, Chine, Japon, Inde, Brésil, Iran et Afrique du Nord. Et, bien sûr, j'ai plaisir à leur rendre visite et à suivre les progrès accomplis. A cette liste j'ajoute l'Espagne qui est aujourd'hui un centre mathématique florissant, alors qu'en 1974, lors de ma première visite, les activités mathématiques y étaient fort réduites.

Je suis très reconnaissant aux organisateurs d'avoir conçu un programme à l'image de cet arc-en-ciel humain. J'ai remarqué avec une immense satisfaction qu'il y avait, parmi les conférenciers, un éminent représentant de mes élèves (Pierre-Louis Lions), plusieurs petits-enfants (Amandine Aftalion, Fabrice Bethuel, François Hamel et Franck Pacard), et même une charmante arrière-petite-fille (Sylvia Serfaty).

On me demande souvent comment j'ai fait pour élever une telle famille. Je réponds comme une mère de famille nombreuse: les trois premiers sont les plus difficiles, ensuite les grands aident les petits... Bernard Larrouturou, qui nous honore de sa présence, se souvient certainement de cette année record où mon cours de 3eme cycle attirait près d'une centaine d'étudiants. Vous imaginez aisément les queues à la porte de mon bureau! Il y a quelques jours, à Gaeta, l'un des "anciens" a fait une confidence publique qui m'a touché: "ça n'était pas désagréable d'attendre des heures devant sa porte – même quand il n'était pas là – car on rencontrait souvent des gens intéressants avec lesquels on pouvait démarrer une collaboration!".

Je suis ravi de voir parmi nous de nombreux amis européens qui me sont chers. Je pense, en particulier, à Michiel Bertsch qui anime depuis Rome, avec grand dévouement, un réseau de l'UE auquel je suis lié, à Bert Peletier de Hollande, à Jean Mawhin de Belgique, à Ildefonso Diaz d'Espagne, à Moshe Marcus d'Israel, avec lesquels nous avons des rapports quotidiens, entre autres, sur le plan d'échanges d'étudiants et de chercheurs.

Je voudrais dire toute mon affection et ma reconnaissance à tous les organisateurs – en particulier à ceux qui ont porté le fardeau sur leurs épaules au fil de longs mois.

Dire à Henri Berestycki combien sa présence rayonnante au laboratoire, à mes côtés, pendant une douzaine d'années a été une source d'énergie. Combien sa rigueur, son intégrité, sa recherche infatigable d'excellence, ses relations internationales, son ouverture aux applications, son dévouement à la nébuleuse de jeunes qui gravitent autour de lui, m'impressionnent.

Dire à Jean-Michel Coron combien je suis fier des travaux de recherche effectués en collaboration au fil d'une dizaine d'années. Cette période a été l'une des plus inspirantes de ma vie de chercheur. Et j'ai un immense plaisir à poursuivre cette collaboration par l'intermédiaire des jeunes superstars qu'il a formées et entraînées dans son sillage.

A Laurent Véron dont j'apprécie la passion et la force de persévérance en recherche. La discrétion et la générosité de son amitié me sont très précieuses.

A Myriam Comte dont le sourire et la bonne humeur illuminent notre Laboratoire. L'arbre généalogique qu'elle a confectionné avec patience depuis des mois, me comble de joie. Comme vous pouvez le constater ce chef-d'œuvre ressemble plus à une forêt touffue qu'à un arbre!

A Didier Smets dont la gentillesse et la serviabilité sont légendaires, et qui s'est dépensé sans compter – car il ne sait pas refuser un service – pour l'organisation matérielle de ce colloque.

Monsieur le Président de l'Université, je forme des vœux fervents pour que cette maison continue à être un espace convivial de liberté et de rencontres avec l'ensemble de la communauté scientifique du monde, sans exclusion.

Le mot de la fin est pour mon épouse. Je voudrais partager avec toi, Michal, et avec vous tous un souvenir douloureux qui me hante. Il y a une quinzaine d'années, nous célébrions le 60eme anniversaire du très regretté Jacques-Louis Lions. Peut-être certains d'entre vous se souviennent de cet événement mémorable. Au cours de la réception J.-L. Lions a pris la parole et à la fin de son allocution il s'est tourné

vers Andrée, sa femme, en exprimant tout son amour et sa gratitude "à celle qui attend avec patience (ou impatience ?) que mes affaires se calment". Comme vous le savez, les "affaires" de J.-L. Lions ne se sont jamais calmées. Eh bien, ce que je vous souhaite, à Michal et à moi, c'est d'avoir la force de décider un jour – pas immédiatement, n'ayez crainte! – que les affaires *doivent* se calmer. Je le ferai avec d'autant plus de sérénité que je sais que la relève est amplement assurée. Je dirai alors à Michal, en empruntant les paroles au Cantique des Cantiques, le plus beau chapitre de la Bible:

"Moi, je suis à ma bien-aimée et ma bien-aimée est à moi." "Viens ma bien-aimée, sortons dans les champs, passons la nuit dans les buissons. De bon matin nous irons dans les vignobles, nous verrons si les vignes fleurissent. Si les bourgeons se sont ouverts, si les grenadiers ont des fleurs. Les mandragores répandent leur parfum. A nos portes il y a toutes sortes de fruits exquis, des nouveaux et des anciens que j'ai gardés pour toi, ma bien-aimée."

**Speech given by Michal Govrin on 22 June 2004 at the
Université Pierre et Marie Curie at the reception in honor of
Haïm Brezis's 60th birthday**

Good evening,

I am happy to be here tonight next to Haïm, and amongst you all. I have arrived to the ceremony directly from the airport, from our home in Jerusalem. And already the way I joined you, confirms the famous theorem of "The Traveling Brezis". In truth, the first time my interest in Haïm was aroused, was in his absence. It was during a visit to his parents' home, Becky and Mico, Rivka and Yaakov Brezis, in 28 rue Berthollet. His mother, Becky, spoke with love and pride about her eldest son, his accomplishments, the prizes he was awarded, and apologized that he is not here because he's traveling. Away? Traveling? Immediately my curiosity arose along with a desire to meet this absent mathematician. And since then a dimension of the presence of absence or, if you wish, one of a nonlinear differential equation, exists in our life together. My first meeting with Haïm's parents occurred in a special time and space. It was during the festival of *Sukkot*, as we sat in the family *sukkah* which they built on the long, narrow balcony of their apartment in rue Berthollet. So narrow it had room only for a long narrow table with a row of chairs on one side. It was truly a surreal *sukkah*, hovering high above a Parisian street, and yet it was so much in the spirit of what a *sukkah* means in the Jewish tradition: a structure defined as temporary; a reminder of the transience of life, of Jewish life, and inside which one should dwell for seven days each year, during the festival of *Sukkot*. A metaphor still so relevant even today. The temporary *sukkah*, with its open, undefined space, is a metaphor to another one of Haïm's traits. As you all know, and as you have heard in his lecture last Friday, Haïm likes open questions. His mathematical curiosity is attracted by them. Today, in this private public event, one can see how much Haïm's life is surrounded by open problems. Beginning with the way we met. I first heard Haïm's voice on the phone, even if this first conversation did not lead to a meeting. It was during the years that I lived in Paris, studying for my PhD (not in mathematics, but in theatre and literature). Saul Friedlander, the renowned historian, referred me to him, as part of the activities of L'Association des Universitaires et Chercheurs pour la Paix au Moyen-Orient. And from then till today the question of peace in the Middle East, and of the relationship between the Middle East and the West, has remained open. Our first real meeting was in Jerusalem. It was after Haïm read an essay I wrote following my visit to Poland, to my mother's home town, Krakow, and to the death camps. My visit to Poland was almost simultaneous with Haïm's visit to Poland, with a similar interest, so that we claim that we met in Poland. But the meeting itself occurred

in Jerusalem. A meeting in a triangular junction: Poland, Paris and Jerusalem. Our lives in face of this triangle and the echoes of its historical heritage are also still an open question. And beyond the historical dimension, you must imagine to yourselves that the couple life of a mathematician and a writer provokes many open questions, which demand quite a talent for nonlinear equations. And even our shared study of Jewish texts, beyond the joy of common reflection, is laden with what the Jewish tradition calls controversies. In Jewish study, the controversy serves as a source of inspiration and fertilization of thought. And in the spirit of mathematics, which I grasped from Haïm, I learned to appreciate the scientific courage to stand before open questions, in theory, and no less in social and historical responsibilities. And especially the courage to face them with freedom of thought, with a force of invention and innovation, without surrendering to preconceptions or stereotypes. Thanks to my life with Haïm, I had the opportunity to meet the family of mathematicians, and many of them became my friends too, all over the world – a very special privilege. Many of them are present here today, and I salute them all personally. From Jerusalem I also bring you congratulations from our two daughters, Rachel-Shlomit and Miriam Rivka (Mirika). Rachel-Shlomit is completing this week her first year at the Hebrew University, in the *Amirim* Honors program and in psychology, and Mirika, a student at the Hebrew University High School, will pass this upcoming Friday her Hebrew Linguistics matriculation exam. And this is also symbolic, as Becky, Haïm's mother, founded and was the principal of the "Talmud Torah", the Sunday Hebrew School at the synagogue of rue Vauquelin in Paris, and according to Haïm, she taught him to read and write in Hebrew long before he was exposed to French. Our two daughters, like Haïm, are diligent. This virtue is the Eros of Jewish studies, as you may read in Haïm's book "Un mathématicien juif". But for me, this is first and foremost the legacy of Mico, Yaakov Brezis, Haïm's father. In his fur shop, on the corner of rue Claude Bernard and rue Vauquelin, Mico used to spend many hours processing the furs, preparing them for use, and sowing them. Many people used to visit him in the workshop corner of the store. Members of the community would stand beside him, talk to him, open their hearts before him. Because, even if his official title was the synagogue treasurer, Mico in fact fulfilled the role of the community's "hidden rabbi". I am very moved to be here today, to see Haïm surrounded by his friends, colleagues, students and grand-students. I extend my heart-felt thanks to the conference organizers, to each and every one, for their effort and devotion. And to you, Haïm, my personal greetings. I hope that this conference – where so many people disclosed their feelings towards you and your influence on them – will provide you with a self-portrait, with all its richness and complexity. A portrait reflecting your contradictions and vitality, as powerful as the world of contradictions and might, described by your beloved *Zohar* – The Book of Splendor. So now, at the age of sixty, you could turn to the fascinating open question: Who is Haïm Brezis? The mathematician, the man, with sentiment and sensitivity. With opinions, soft and harsh. Solitary and man of the world. Devoted father and husband, and traveler to the ends of the world. Yes, an open and a moving question, and at its heart the constant aspiration to be a *mentsch*, this Yiddish term for a 'real person'. And my own special blessing for you, Haïm, may you have many years of health and creativity, of finding some solutions and even more so of opening many new open questions – in life and in creation, as a father and as a partner, and in the great adventure of our life together. And to

all the guests at this conference, colleagues, former students, researchers and future students, I wish you a fruitful and prolific conference, and many years of creation.

In celebration of Haïm Brezis's 60th birthday, a conference was held at the École Polytechnique in Paris, with a program testifying to Brezis's wide-ranging influence on nonlinear analysis and partial differential equations. The articles in this volume are primarily from that conference. They present a rare view of the state of the art of many aspects of nonlinear PDEs, as well as describe new directions that are being opened up in this field. The articles, written by mathematicians at the center of current developments, provide somewhat more personal views of the important developments and challenges.

In his own work, Brezis has been a seminal influence in many important areas: critical growth in semi-linear equations, variational analysis of functionals in borderline compactness cases, vector valued equations, the Ginzburg-Landau theory, as well as generalized degree theory and fine properties of Sobolev spaces. This same breadth is reflected in the mathematics in this collection.

Researchers in nonlinear partial differential equations will find much of interest in this volume.

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