CONTEMPORARY MATHEMATICS

465

Curves and Abelian Varieties

International Conference March 30–April 2, 2007 University of Georgia Athens, Georgia

> Valery Alexeev Arnaud Beauville C. Herbert Clemens Elham Izadi Editors



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Preface

This is a volume of proceedings of the conference organized by Valery Alexeev and Elham Izadi at the University of Georgia on March 30 - April 2, 2007. The conference was centered on a circle of questions about curves, abelian varieties, and Hodge structures. The talks represented many sides of the subject: classical aspects, varieties with group action and their moduli, Hodge structures and normal functions, log schemes of Illusie-Fontaine-Kato, and tropical geometry. Among blooming azaleas, about 70 participants from all over United States, France, Italy, Germany and Brasil met, which included established researchers, young postdocs, and many students.

The conference was held almost 30 years after another well known Georgia conference was organized by our esteemed colleague Roy Smith. Thus, it was our particular pleasure to dedicate this conference to his 65th birthday. Roy Smith's contributions to mathematics at the University of Georgia are many, and widely appreciated. But of particular importance, as well as being the rationale of this volume, is his central role in establishing algebraic geometry as one the fields of mathematics for which the University of Georgia is known and respected.

Thanks are due to all the participants, to the National Science Foundation for major funding, as well as to the UGA President's Venture Fund, the UGA Department of Mathematics, the Office of Vice President for Research, and the UGA Graduate School.

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- (1) Arnaud Beauville (University of Nice, France). Algebraic cycles on Jacobian varieties.
- (2) Michel Brion (University of Grenoble, France). Actions of non-affine algebraic groups.
- (3) Lucia Caporaso (University of Rome-3, Italy). Abel maps and theta divisors for stable curves.
- (4) Sebastian Casalaina-Martin (Harvard University). Prym varieties and the Schottky problem for cubic threefolds.
- (5) Herb Clemens (The Ohio State University). Making the Hodge problem "more geometric".
- (6) Phillip A. Griffiths (The Institute for Advanced Study, Princeton). Degeneration of Abel Jacobi mappings and Néron models for intermediate jacobians.
- (7) Samuel Grushevsky (Princeton University). Intersection numbers of divisors on the moduli space of abelian varieties.
- (8) Matt Kerr (University of Chicago). Algebraic K-theory of toric hypersurfaces.
- (9) Igor Krichever (Columbia University). Integrable linear equations of the soliton theory and Riemann-Schottky type problems.
- (10) Herbert Lange (Erlangen, Germany). Polarizations of Prym varieties via abelianization.
- (11) David Lehavi (University of Michigan). There are no projective surfaces in M_4 .
- (12) Alina Marian (Yale University). The level-rank duality for nonabelian theta functions.
- (13) Martin Olsson (University of California, Berkeley). A higher dimensional version of generalized elliptic curves.
- (14) Giuseppe Pareschi (University of Rome-2, Italy). Generic vanishing and Fourier-Mukai transforms.
- (15) Gregory Pearlstein (Michigan State University). On the zero locus of a normal function.
- (16) Ravi Vakil (Stanford University). Murphy's Law in algebraic geometry: Badly-behaved moduli spaces.
- (17) Alessandro Verra (University of Rome-3, Italy). On the Prym moduli spaces of curves in low genus.
- (18) Ilia Zharkov (Harvard University). Tropical curves and their Jacobians.

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This book is devoted to recent progress in the study of curves and abelian varieties. It discusses both classical aspects of this deep and beautiful subject as well as two important new developments, tropical geometry and the theory of log schemes.

In addition to original research articles, this book contains three surveys devoted to singularities of theta divisors, of compactified Jacobians of singular curves, and of "strange duality" among moduli spaces of vector bundles on algebraic varieties.

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