

# CONTEMPORARY MATHEMATICS

475

## Singularities II Geometric and Topological Aspects

International Conference  
in Honor of the 60th Birthday of Lê Dũng Tráng  
January 8–26, 2007  
Cuernavaca, Mexico

Jean-Paul Brasselet  
José Luis Cisneros-Molina  
David Massey  
José Seade  
Bernard Teissier  
Editors



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To Lê Dũng Tráng in celebration of his first 60 years.

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## Preface

These are the Proceedings of the meeting “School and Workshop on the Geometry and Topology of Singularities”, held in Cuernavaca, Mexico, from January 8th to 26th of 2007, in celebration of the 60th Birthday Anniversary of Lê Dũng Tráng.

This meeting lasted three weeks. During the first week, the activities consisted of 6 elementary courses. In the second week, there were 4 more-advanced courses and 13 Invited Lectures. In the third week, there was a course by Professor Heisuke Hironaka, 23 Invited Lectures, and a Poster Session.

This meeting was part of a program launched and supported by the Abdus Salam ICTP, in Trieste, Italy, and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) of Brazil, with the aim of enhancing science in the developing world. The meeting was organized by the Cuernavaca Unit of the Instituto de Matemáticas of Universidad Nacional Autónoma de México (UNAM), with significant support from the Clay Mathematics Institute (U.S.A.) and Consejo Nacional de Ciencia y Tecnología (CONACYT) of Mexico. The articles presented here are all research articles, either written by invitation, or presented by Invited Speakers.

Singularity theory is a meeting place of many disparate areas of mathematics, where different types of ideas, techniques, and results merge together. The modern theory of singularities dates back to the 1960s, with the pioneering work of René Thom, Heisuke Hironaka, Egbert Brieskorn, Oscar Zariski and many other renowned mathematicians. Lê Dũng Tráng was one of the privileged “sons” of that golden generation.

For more than three decades, Lê Dũng Tráng has been a pillar of singularity theory. His early works with Bernard Teissier, Helmut Hamm, C. P. Ramanujam, Kyoji Saito, and others are landmarks in singularity theory, and Lê has continued to be a mathematical leader ever since. He has published more than 110 research articles, he has had 18 Ph.D. students, and he has been a mentor to an entire generation of researchers.

In addition to contributing greatly to our knowledge of singularities, Lê Dũng Tráng has contributed extensively in other ways to the mathematical community. His mentoring, collaborating, organizing of conferences and social events, editorial positions, and international efforts have had, and continue to have, a profound effect on mathematics and mathematicians throughout the world.

Lê Dũng Tráng has worked in all areas of singularities, including algebraic, analytic, geometric, and topological aspects of singularities. Thus, not surprisingly, although the title of the meeting referred to geometry and topology, we actually

had a vast participation of mathematicians coming from many different viewpoints of singularity theory.

Therefore, we decided to publish these proceedings in two independent volumes, according to a -sometimes artificial- judgment by the editors as to what are the primary topics and techniques of each article. One volume focuses on algebraic and analytic aspects of singularity theory; the other focuses on geometric and topological viewpoints. Even though such a classification may seem arbitrary in some cases, this splitting of the proceedings of the “Lê-Fest” should help the reader to find more easily the articles that are most relevant to him/her at a given moment.

The Scientific Committee was composed of Jean-Paul Brasselet (Institut de Mathématiques de Luminy-CNRS, Luminy, France), David Massey (Department of Mathematics, Northeastern University, Boston, USA), José Seade (Instituto de Matemáticas Unidad Cuernavaca, UNAM, Cuernavaca, Mexico) and Bernard Teissier (Institut Mathématique de Jussieu, CNRS-Paris 6-Paris 7, Paris, France).

There was also an Organizing Committee, composed of the local organizers Haydée Aguilar, Fuensanta Aroca, José Luis Cisneros-Molina, Elsa Puente and Jawad Snoussi, (all from Instituto de Matemáticas, UNAM, Unidad Cuernavaca, Mexico), and David Alexandre Ellwood (Clay Mathematics Institute, USA) and Marcio Soares, (Universidade Federal de Minas Gerais, Brazil), who did a magnificent job of organizing a very strong participation of mathematicians from Brazil, mostly from the São Carlos school. All of the local organizers did tremendous work, and the meeting was so full of good mathematics, friendship, and happiness, that we are certain this will leave a mark in the heart of each participant, and will be a most stimulating remembrance for the many young mathematicians who participated in the meeting.

The Editors

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This is the second part of the Proceedings of the meeting “School and Workshop on the Geometry and Topology of Singularities”, held in Cuernavaca, Mexico, from January 8th to 26th of 2007, in celebration of the 60th Birthday of Lê Dũng Tráng.

This volume contains fourteen cutting-edge research articles on geometric and topological aspects of singularities of spaces and maps. By reading this volume, and the accompanying volume on algebraic and analytic aspects of singularities, the reader should gain an appreciation for the depth, breadth, and beauty of the subject, and also find a rich source of questions and problems for future study.

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