Communicating Mathematics
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Preface

As anyone who has met him knows, Joseph Gallian is unique. It is not just his achievements that make him unique, even though there is a long list of them: his highly successful summer research experience for undergraduates (REU), started in the days when most people believed undergraduates incapable of doing mathematics research; his textbooks *Contemporary Abstract Algebra* and *For All Practical Purposes: Mathematical Literacy in Today’s World* (co-author); his co-directorship of Project NExT; his numerous research articles; his countless awards and honors for teaching and service; and his legendary mathematics lectures that are accessible to broad audiences. Rather, Gallian is unique because of his dynamic personality, his infectious enthusiasm, and his love for mathematics, all of which have combined to inspire research, encourage networking among mathematicians, and communicate mathematics effectively.

The value of Gallian’s contribution to the mathematical community cannot be overstated. The star researcher may contribute to mathematics by writing seminal papers, and the gifted expositor may contribute to mathematics by writing classic textbooks, but the one who enables dozens of young mathematicians to become star researchers and gifted expositors contributes far more. Gallian is the enabler par excellence. He has an egalitarian philosophy that mathematics is not an exclusive, elitist club, but is open to all. It is no accident that many alumni of the Duluth REU are not only outstanding researchers but also excellent communicators, because clear exposition is critical to breaking down the barriers that stand in the way of mathematical progress.

To celebrate the Gallian vision of mathematics, the conference *Communicating Mathematics* was held at the University of Minnesota Duluth in July 2007. This conference marked the 30th anniversary of the Duluth REU, as well as Gallian’s 65th birthday. The overarching objective of the gathering was to inspire research productivity and enthusiasm among mathematicians at all stages of their careers. The conference program consisted of colloquium-style lectures delivered by experts to communicate current research ideas in a wide variety of mathematical fields, together with shorter contributed talks, and many opportunities for participants to develop and renew meaningful collaborations.

In keeping with Gallian’s uniqueness, we have tried to make the volume you are now holding unique, and more than just another conference proceedings. Some articles are research articles and others are purely expository, but in all cases we have pushed the authors to produce works of high expository quality. The papers are intended to be accessible to a general mathematics audience, including first-year or second-year graduate students. We hope that this volume will inspire research productivity and celebrate fine exposition, and will be especially useful for junior
researchers as well as mathematicians considering a change in research area. We also expect that it will be used more generally by mathematicians looking to enrich themselves and their research programs by looking at the problems and techniques used in other areas. We would especially like to draw the reader’s attention to the biography of Gallian written by Aparna Higgins, one of Gallian’s close collaborators.

In short, if the reader of this volume catches some of the excitement of the Gallian legacy and is inspired to create and communicate mathematics effectively, then we will have succeeded in our aims. Enjoy!

Acknowledgements. The editors appreciate the valuable contributions of many people in preparing and assembling this volume. In particular, they thank Brett Groehler of the University of Minnesota Duluth for the photograph of Gallian; Geir Helleloid for editorial assistance; and Christine Thivierge of the AMS for assistance and guidance throughout the publication process.
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This volume contains the proceedings of a conference held in July, 2007 at the University of Minnesota, Duluth, in honor of Joseph A. Gallian’s 65th birthday and the 30th anniversary of the Duluth Research Experience for Undergraduates.

In keeping with Gallian’s extraordinary expository ability and broad mathematical interests, the articles in this volume span a wide variety of mathematical topics, including algebraic topology, combinatorics, design theory, forcing, game theory, geometry, graph theory, group theory, optimization, and probability.

Some of the papers are purely expository while others are research articles. The papers are intended to be accessible to a general mathematics audience, including first-year or second-year graduate students. This volume should be especially useful for mathematicians seeking a new research area, as well as those looking to enrich themselves and their research programs by learning about problems and techniques used in other areas of mathematics.