CONTEMPORARY MATHEMATICS

492

Mathematics, Developmental Biology and Tumour Growth

UIMP–RSME Lluis A. Santaló Summer School
September 11-15, 2006
Universidad Internacional Menéndez Pelayo,
Santander, Spain

Fernando Giráldez
Miguel A. Herrero
Editors

American Mathematical Society
Real Sociedad Matemática Española

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Preface

This volume contains a number of selected survey papers on the topics presented and discussed at the "Lluis A. Santaló Summer School", that was devoted to Mathematics of Development and Cancer. The School was held in Santander (Spain) on September 11–15, 2006, as part of the activities of the Universidad Internacional Menéndez Pelayo (UIMP), in collaboration with the Real Sociedad Matemática Española (RSME). Lecturers came from different scientific fields, including Biology, Mathematics, Medicine and Physics. They were selected in an attempt to present an outline of ongoing research in selected areas of mathematics and biology, in a manner that could be widely accessible to an audience consisting mainly of advanced undergraduates and graduate students on Mathematics. These were thus given a front seat at the research currently done by a number of groups worldwide working in Biology and Mathematics. The course was centred on Developmental Biology and Tumour Growth. These are topics where Mathematics is increasingly being used as a new and powerful technique to gain new insights. Mathematics is in its turn receiving a significant scientific pay-off in the form of new and challenging mathematical problems to be added to their own ones.

For instance, Developmental Biology is an area of basic research in Biology and Medicine that has fascinated mankind since the earliest recorded scientific thought. Developmental Biology, which has experienced a great impulse during last years, deals with the basic problem of understanding the unfolding of utterly complex living structures "from egg to embryo". In this manner, it has been seen to raise a number of challenging quantitative problems that immediately appeal at the imagination of mathematicians. This is the case of some central problems in Biology related to pattern formation, where the issue of how space and time evolution is first coded in the genome, and then set in action during development, has generated a great interest in the use of mathematical tools.

The first part of this volume contains three views on different aspects of Developmental Biology as seen with the eyes of biologists interested in incorporating Mathematics to their technical tools. More precisely, Alsina et al. summarise the state of the art in Developmental Biology by describing some of the basic questions in the field, and by commenting on some examples of formal approaches to the specific problem of patterning in development. Arques and Torres describe the problem of how three dimensional patterns are established in the limbs of vertebrates, and show how an interesting model on mesenchymal compartments formation can be derived from a clever clonal analysis of cell lineages. Finally, Henrique and Fior discuss on one of the key signalling pathways in development, the Notch pathway, a subject on which a good deal of work is being currently done both by biologists and mathematicians.

On the other hand, Cancer Research is a major priority in health care and a challenge to the scientific community as a whole. Cancer is a complex biological process that raises issues in Genetics, Cell Differentiation, Environmental Sciences, Immunology, Pharmacology, Physics,... to mention but a few active fields of scientific endeavour. Interdisciplinary research is thus a growing need of mathematicians working along physicians and biologists in order to develop models and new therapeutic approaches.

In this volume, the reader will find contributions by Alarcón, Cappuccio, Herrero and Núñez, and Galle and Preziosi , all of them dealing with the mathematical modelling and subsequent analysis of problems related to Tumour Growth. More precisely, Alarcón provides an updated account of current views on angiogenesis, the formation of new blood vessels from a preexisting vasculature, which is known to play a key role in many types of tumour progression. Cappuccio et al. give an overview of modelling issues arising in radiotherapy, a commonly used technique to treat a number of malignant (and also benign) tumours. On their turn, Galle and Preziosi present a comprehensive picture of tumour growth insisting on individual cell-aspects and their relation to a multiphase -fluid flow picture of that process.

The book is addressed to PhD students and advanced undergraduate students in sciences willing to start their research in the exciting, and as yet unchartered, interface among Biology and Mathematics. There is a widely perceived interest in finding common spaces for discussion between mathematical and biological sciences, and this volume is expected to provide a contribution to that goal. The aim was at presenting an updated view on some basic problems of Biology, and to illustrate how Mathematics may help to improve knowledge on some of them. It goes without saying that the choice of topics has no claim to be exhaustive, and that the authors are fully aware that many interesting subjects have been left out of the selection herein made.

The editors wish to thank the RSME for giving them the opportunity to plan the School from which this work has unfolded. Our thanks also go to the UIMP, which provided excellent organization, and outstanding facilities, to organise the series of lectures which are at the origin of this book.

Fernando Giráldez and Miguel A. Herrero

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Developmental biology and tumour growth are two important areas of current research where mathematics increasingly provides powerful new techniques and insights. The unfolding complexity of living structures from egg to embryo gives rise to a number of difficult quantitative problems that are ripe for mathematical models and analysis. Understanding this early development process involves the study of pattern formation, which mathematicians view through the lens of dynamical systems. This book addresses several issues in developmental biology, including Notch signalling pathway integration and mesenchymal compartment formation.

Tumour growth is one of the primary challenges of cancer research. Its study requires interdisciplinary approaches involving the close collaboration of mathematicians, biologists and physicians. The summer school addressed angiogenesis, modelling issues arising in radiotherapy, and tumour growth viewed from the individual cell and the relation to a multiphase-fluid flow picture of that process.

This book is suitable for researchers, graduate students, and advanced undergraduates interested in mathematical methods of developmental biology or tumour growth.

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