

CONTEMPORARY MATHEMATICS

493

Quadratic Forms—Algebra, Arithmetic, and Geometry

Algebraic and Arithmetic Theory of Quadratic Forms,
December 13–19, 2007
Frutillar, Chile

Ricardo Baeza
Wai Kiu Chan
Detlev W. Hoffmann
Rainer Schulze-Pillot
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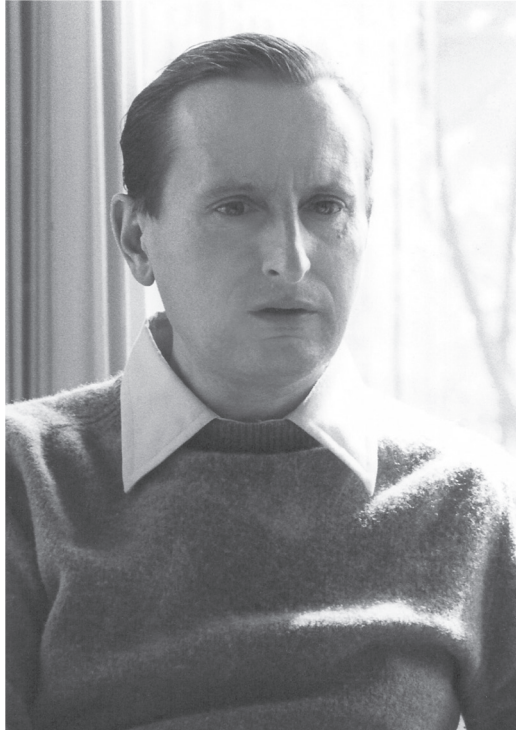
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In Memoriam
Martin Kneser (1928–2004)



Photograph by Winfried Scharlau Oberwolfach, 1974.

Martin Kneser was born on January 21, 1928 in Greifswald and died February 16, 2004 in Göttingen. He received his doctoral degree in Berlin (1950) and the habilitation in Heidelberg (1953). Subsequently he held professorships in Saarbrücken (1958), München (1959) and Göttingen (1963), where he retired in 1993. He had a strong influence on the development of the theory of quadratic forms, the theory of algebraic groups, and the connections between both; the article “Martin Kneser’s work on quadratic forms and algebraic groups” by Rudolf Scharlau in this volume gives details. The quadratic forms community as a whole and the editors in particular owe a great intellectual and personal debt to him. We dedicate this volume to the memory of Martin Kneser.

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Preface

As a sequel to the International Conference on the Algebraic and Arithmetic Theory of Quadratic Forms in Talca and Pucón (Chile) 2002, whose proceedings were published as volume 344 of this series, another conference with the same title was held in December 2007 in Frutillar in the southern part of Chile. This volume contains the proceedings of the meeting, which was attended by 90 participants from North and South America, Asia and Europe.

The talks at the conference covered the full range of topics that constitute the theory of quadratic forms, from the algebraic theory with its connections to algebraic K -theory, (real) algebraic geometry, and Galois cohomology to the arithmetic theory with its connections to analytic number theory, geometric problems, optimization, and coding theory. The papers in this proceedings volume reflect the diversity of the talks given at the conference and range from survey articles, serving as an introduction to important results of recent years, to articles presenting new research results and open problems.

The editors gratefully acknowledge financial support for the conference from the following sources:

- (1) Proyecto Anillo, ACT 05, PBCT, Conicyt
- (2) Universidad de Talca, Proyecto "Reticulados y Ecuaciones"
- (3) Royal Society International Joint Project "Differential Forms and their Applications to Quadratic and Bilinear Forms"

The conference would not have been possible without the great efforts by the local organizing committee, headed by M. I. Icaza and M. O'Ryan, that took care of all the logistic and organizational problems and the many little worries of the participants. The editors Chan, Hoffmann and Schulze-Pillot owe much debt to Ricardo Baeza for bringing this meeting into existence and coordinating the scientific program and the editing process of this volume. All four editors thank the AMS for publishing this volume and the staff of the AMS for all the support in the editing process. Last but not least we thank all the participants for traveling long distances and for their varied contributions that made the meeting such a scientific success.

Ricardo Baeza
Wai Kiu Chan
Detlev W. Hoffmann
Rainer Schulze-Pillot

February 2009

List of Participants

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This volume presents a collection of articles that are based on talks delivered at the International Conference on the Algebraic and Arithmetic Theory of Quadratic Forms held in Frutillar, Chile in December 2007.

The theory of quadratic forms is closely connected with a broad spectrum of areas in algebra and number theory. The articles in this volume deal mainly with questions from the algebraic, geometric, arithmetic, and analytic theory of quadratic forms, and related questions in algebraic group theory and algebraic geometry.

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