

CONTEMPORARY MATHEMATICS

500

Spectral and Scattering Theory for Quantum Magnetic Systems

July 7–11, 2008
CIRM, Luminy
Marseilles, France

Philippe Briet
François Germinet
Georgi Raikov
Editors



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2000 *Mathematics Subject Classification*. Primary 81Q10, 81V10, 35J10, 82B44, 60H25, 47B80, 81Q70, 35P20, 35P25.

Library of Congress Cataloging-in-Publication Data

Spectral and scattering theory for quantum magnetic systems, July 7–11, 2008, CIRM, Luminy, Marseilles, France / Philippe Briet, François Germinet, Georgi Raikov, editors.

p. cm. — (Contemporary mathematics ; v. 500)

Includes bibliographical references.

ISBN 978-0-8218-4744-2 (alk. paper)

1. Quantum theory—Mathematical models—Congresses. 2. Quantum scattering—Congresses. 3. Mathematical physics—Congresses. I. Briet, Philippe, 1954– II. Germinet, François, 1970– III. Raikov, Georgi, 1954– IV. Centre international de rencontres mathématiques (France)

QC173.96.S64 2009

538.01'5118—dc22

2009022736

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10 9 8 7 6 5 4 3 2 1 14 13 12 11 10 09

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Preface

This volume contains the Proceedings of the Conference “Spectral and Scattering Theory for Quantum Magnetic Systems” which took place at CIRM, Luminy, France, in July 2008. The main purpose of this conference was to bring together a number of specialists in the mathematical modelling of magnetic phenomena in quantum mechanics, to mark the recent progress as well as to outline the future development in this area.

Quantum magnetic systems are studied by various mathematical “schools” or “communities”, which investigate different mathematical aspects of related problems, developing their specific appropriate techniques. The meeting was meant to give the researchers from these communities the opportunity to discuss and exchange ideas, as well as to attract young scientists to this area of research.

There were two mini-courses given by Bernard Helffer and Hajo Leschke, twenty six invited talks given by Rafael Benguria, Vincent Bruneau, Horia Cornean, Mouez Dimassi, Daniel Elton, Maria Esteban, Claudio Fernández, Soeren Fournais, Rupert Frank, Gian Michele Graf, Benoît Grébert, Ira Herbst, Peter Hislop, Victor Ivrii, Abel Klein, Frédéric Klopp, Yuri Kordyukov, Marius Măntoiu, Takuya Mine, Vesselin Petkov, Alexander Pushnitski, Serge Richard, Grigori Rozenblum, Robert Seiringer, Eric Soccorsi, Rafael Tiedra de Aldecoa, and five students’ mini-talks given by Mathieu Beau, Nicolas Dombrowski, Abdallah Khochman, Max Lein, and Nicolas Raymond.

This volume contains a number of original results presented by some of the invited speakers, and also surveys on recent advances in the mathematical theory of quantum magnetic Hamiltonians. Most of the talks at the conference, as well as the articles in this volume, have been dedicated to one of the following topics:

- Spectral and scattering theory for magnetic Schrödinger operators;
- Magnetic Pauli and Dirac operators;
- Magnetic operators on manifolds;
- Microlocal analysis of magnetic Hamiltonians;
- Random Schrödinger operators and quantum Hall effect;
- Ginsburg-Landau equation, supraconductivity, magnetic bottles;
- Bose-Einstein condensate, Gross-Pitaevski equation;
- Magnetic Lieb-Thirring inequalities, stability of matter.

We are grateful to the International Center for Mathematical Meetings (CIRM) for the hospitality, for the financial and logistic support which made this meeting

possible. We are also grateful to the rest of our sponsors: the International Association of Mathematical Physics, Centre de Physique Théorique Marseille, Université de Cergy-Pontoise, Université de la Méditerranée, Fédération de Recherche des Unités de Mathématiques de Marseille, CNRS Département MPPU, GDRE Mathematics and Quantum Physics, Ville de Marseille, Région Provence-Alpes-Côte d'Azur, Conseil Général des Bouches du Rhône.

We would like to thank our mini-course lecturers Bernard Helffer and Hajo Leschke for the excellent job they did, our invited speakers, the students who gave mini-talks, and all the people who came to this meeting, for their active participation.

Finally, we would like to thank the authors who contributed to this volume, the referees for their professional work, Lilia Simeonova for handling manuscripts and referee reports, the editorial board of the AMS for giving us the possibility to publish these proceedings, and especially Christine Thivierge for her assistance.

Ph. Briet,
F. Germinet,
G. Raikov

List of Participants

Hamadi BAKLOUTI
Faculté des Sciences de Sfax
Tunisia
h_baklouti@yahoo.com

Yves DERMENJIAN
Université de Provence LATP
France
dermenji@gyptis.univ-mrs.fr

Mathieu BEAU
Université de la Méditerranée, CPT
France
beau@cpt.univ-mrs.fr

Mouez DIMASSI
Université Paris-Nord
France
dimassi@math.univ-paris13.fr

Rafael BENGURIA
Pontificia Universidad Católica de Chile
Chile
rbenguri@fis.puc.cl

Nicolas DOMBROWSKI
Université Cergy-Pontoise
France
nicolas.dombrowski@u-cergy.fr

François BENTOSELA
Université de la Méditerranée, CPT
France
francois.bentosela@univmed.fr

Daniel ELTON
Lancaster University
United Kingdom
d.m.elton@lancaster.ac.uk

Philippe BRIET
Université Sud Toulon-Var, CPT
France
briet@univ-tln.fr

Maria J. ESTEBAN
Université Paris-Dauphine
France
esteban@ceremade.dauphine.fr

Vincent BRUNEAU
Université Bordeaux 1
France
Vincent.Bruneau@math.u-bordeaux1.fr

Claudio FERNANDEZ
Pontificia Universidad Católica de Chile
Chile
cfernand@mat.puc.cl

Jean-Michel COMBES
Université Sud Toulon-Var, CPT
France
combes@cpt.univ-mrs.fr

Soeren FOURNAIS
University of Aarhus
Denmark
fournais@imf.au.dk

Horia CORNEAN
Aalborg University
Denmark
cornean@math.aau.dk

Rupert FRANK
Princeton University
USA
rlfrank@math.princeton.edu

François GERMINET
 Université Cergy-Pontoise
 France
 germinet@math.u-cergy.fr

Akira IWATSUKA
 Kyoto Institute of Technology
 Japan
 iwatsuka@kit.ac.jp

Fatma GHRIBI
 Faculté des Sciences-Monastir
 Tunisia
 Fatma.Ghribi@fsm.rnu.tn

Abdallah KHOCHMAN
 Université Bordeaux 1, France
 abdallah.khochman@math.u-
 bordeaux1.fr

Gian Michele GRAF,
 Institut für Theoretische Physik, Zürich
 Switzerland
 gmgraf@itp.phys.ethz.ch

Abel KLEIN
 UC-Irvine
 USA
 aklein@math.uci.edu

Benoît GRÉBERT,
 Université de Nantes
 France
 benoit.grebert@univ-nantes.fr

Frédéric KLOPP
 Université Paris-Nord, LAGA
 France
 klopp@math.univ-paris13.fr

Bernard HELFFER
 Université Paris-Sud
 France
 Bernard.Helffer@math.u-psud.fr

Yuri KORDYUKOV
 Russian Academy of Sciences
 Russia
 yurikor@matem.anrb.ru

Rainer HEMPEL
 TU-Braunschweig
 Germany
 r.hempel@tu-braunschweig.de

Stanislas KUPIN
 CMI - Université de Provence
 France
 kupin@cmi.univ-mrs.fr

Ira HERBST
 University of Virginia
 USA
 iwh@virginia.edu

Max LEIN
 TU-München
 Germany
 lein@ma.tum.de

Peter HISLOP,
 University of Kentucky
 USA
 hislop@ms.uky.edu

Hajo LESCHKE
 Universität Erlangen-Nürnberg
 Germany
 Hajo.Leschke@physik.uni-erlangen.de

Wataru ICHINOSE,
 Shinshu University
 Japan
 ichinose@math.shinshu-u.ac.jp

Benoît MANDY
 Université Cergy-Pontoise
 France
 benoit.mandy@u-cergy.fr

Victor IVRII
 University of Toronto
 Canada
 ivrii@math.toronto.edu

Marius MANTOIU
 Universidad de Chile
 Chile
 Marius.Mantoiu@imar.ro

Noureddine MHADEBI
 Faculté des Sciences de Tunis
 Tunisia
 Noureddine.Mhadbi@fst.rnu.tn

Takuya MINE
 Kyoto Institute of Technology
 Japan
 mine@math.kyoto-u.ac.jp

Pablo MIRANDA
 Universidad de Chile
 Chile
 pablitomira@gmail.com

Shu NAKAMURA
 University of Tokyo
 Japan
 shu@ms.u-tokyo.ac.jp

Fumihiko NAKANO
 Kochi University
 Japan
 nakano@math.kochi-u.ac.jp

Yuji NOMURA
 Ehime University
 Japan
 nomura@cs.ehime-u.ac.jp

Konstantin PANKRASHKIN
 Université Paris-Nord, LAGA
 France
 konstantin.pankrashkin@math.u-psud.fr

Mihai PASCU
 Romanian Academy
 Romania
 Mihai.Pascu@imar.ro

Vesselin PETKOV
 Université Bordeaux 1
 France
 petkov@math.u-bordeaux1.fr

Alexander PUSHNITSKI
 King's College-London
 United Kingdom
 alexander.pushnitski@kcl.ac.uk

Georgi RAIKOV
 Pontificia Universidad Católica de Chile
 Chile
 graikov@mat.puc.cl

Nicolas RAYMOND,
 Université Paris-Sud
 France
 nicolas.raymond@math.u-psud.fr

Serge RICHARD
 Université Claude-Bernard
 France
 srichard@math.univ-lyon1.fr

Constanza ROJAS MOLINA
 Université Paris 6
 France
 constanza777@yahoo.com

Grigori ROZENBLIUM
 Chalmers-UT
 Sweden
 grigori@chalmers.se

Baptiste SAVOIE
 Université de la Méditerranée, CPT
 France
 savoie@cpt.univ-mrs.fr

Robert SEIRINGER
 Princeton University
 USA
 rseiring@math.princeton.edu

Éric SOCCORSI
 Université de la Méditerranée, CPT
 France
 soccorsi@cpt.univ-mrs.fr

Olga TCHEBOTAREVA
 UNAM
 Mexico
 olga@lya.fciencias.unam.mx

Rafael TIEDRA DE ALDECOA
 Université de Cergy-Pontoise
 France
 rtiedra@math.u-cergy.fr

Françoise TRUC
Université Joseph-Fourier
France
trucfr@ujf-grenoble.fr

Tomio UMEDA
Himeji Institute of Technology
Japan
umeda@sci.u-hyogo.ac.jp

Dimitri YAFAEV
Université Rennes 1
France
dimitri.yafaev@univ-rennes1.fr

Valentin ZAGREBNOV
Université de la Méditerranée, CPT
France
zagrebno@cpt.univ-mrs.fr

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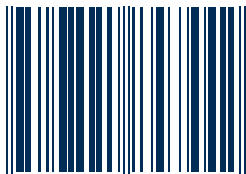
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- Bose–Einstein condensate, Gross–Pitaevski equation
- Magnetic Lieb–Thirring inequalities, stability of matter

ISBN 978-0-8218-4744-2



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