

CONTEMPORARY MATHEMATICS

507

Recent Trends in Orthogonal Polynomials and Approximation Theory

International Workshop in Honor of
Guillermo López Lagomasino's 60th Birthday
September 8–12, 2008
Universidad Carlos III de Madrid
Leganés, Spain

Jorge Arvesú
Francisco Marcellán
Andrei Martínez-Finkelshtein
Editors



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With our best wishes to
Guillermo López Lagomasino
on the occasion of his 60th birthday.

Contents

Preface	ix
Guillermo López Lagomasino: Mathematical life FRANCISCO MARCELLÁN AND ANDREI MARTÍNEZ-FINKELSHTEIN	1
A walk through approximation theory BERNARDO DE LA CALLE YSERN	25
Asymptotic uniqueness of best rational approximants to complex Cauchy transforms in L^2 of the circle LAURENT BARATCHART AND MAXIM YATTSELEV	87
Quadrature rules on the unit circle. A survey. LUIS GARZA AND FRANCISCO MARCELLÁN	113
On the multilinear trigonometric problem of moments ALBERTO IBORT, PABLO LINARES, AND JOSE G. LLAVONA	141
Multiple orthogonal polynomial ensembles ARNO B. J. KUIJLAARS	155
Some equivalent formulations of universality limits in the bulk ELI LEVIN AND DORON S. LUBINSKY	177
Greedy energy points with external fields ABEY LÓPEZ GARCÍA	189
On asymptotic behavior of Heine-Stieltjes and Van Vleck polynomials ANDREI MARTÍNEZ-FINKELSHTEIN AND EVGENII A. RAKHMANOV	209
Remarks on relative asymptotics for general orthogonal polynomials EDWARD B. SAFF	233
Fine structure of the zeros of orthogonal polynomials: A progress report BARRY SIMON	241
A potential-theoretic problem connected with complex orthogonality HERBERT STAHL	255
Orthogonal polynomials and approximation theory: Some open problems WALTER VAN ASSCHE	287

Preface

This volume contains articles by speakers in the International Workshop on Orthogonal Polynomials and Approximation Theory, held on September 8-12, 2008, at the Leganés campus of the Universidad Carlos III de Madrid, in honor of the 60th birthday of Professor Guillermo López Lagomasino. The five-day conference attracted 140 participants from 22 countries. The program of the conference, video, and pdf files of most of the talks can be found at <http://gama.uc3m.es>.

This volume's articles cover a wide range of topics in the theory of Orthogonal Polynomials as well as in Rational Approximation with a special emphasis on their applications in random matrices, integrable systems, and numerical quadrature. The contributors are celebrated researchers in these domains.

A brief account of the scientific contributions by Professor Guillermo López Lagomasino constitutes one of the features of this volume. His activity in the former Soviet Union, in the framework of the group lead by A. A. Gonchar, as well as their connections with the Western groups working in Orthogonal Polynomials and Approximation Theory, fulfils a good historical perspective of the internationalization of these domains in the last 50 years. This is an example of how mathematicians are able to open physical and political barriers and contribute to the improved links between individuals and institutions.

New results and methods are presented in the contributed papers as well as a careful choice of open problems to foster an interest in research in these mathematical areas in the coming years. The reader will find survey presentations, an account of recent developments, and the exposition of new trends in the areas of Orthogonal Polynomials, Special Functions, and Approximation Theory, from a theoretical and an applied perspectives.

As co-organizers of the workshop and editors of this volume it is our happy task to thank those individuals and institutions whose efforts made it possible. First, we acknowledge Ministerio de Educación y Ciencia of Spain (grant MTM2007-31221-E), Proyecto Ingenio Mathematica (i-MATH)(grant C3-0116), Universidad Carlos III de Madrid, Universidad Politécnica de Madrid, Real Sociedad Matemática Española, Sociedad Española de Matemática Aplicada, Ayuntamiento de Leganés, and International Association for the promotion and cooperation with scientists (INTAS) for their financial support. Second, it is a pleasure to thank all the members of the local organization Committee of the Universidad Carlos III de Madrid for the excellent organization of this meeting. Last, but certainly not least, we express our gratitude to the participants of the workshop who made it a memorable

event, to the contributors to this volume, and to Christine Thivierge of the AMS staff for her efficient support in the production of these proceedings.

Jorge Arvesú, Universidad Carlos III de Madrid
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This volume contains invited lectures and selected contributions from the International Workshop on Orthogonal Polynomials and Approximation Theory, held at Universidad Carlos III de Madrid on September 8–12, 2008, and which honored Guillermo López Lagomasino on his 60th birthday.

This book presents the state of the art in the theory of Orthogonal Polynomials and Rational Approximation with a special emphasis on their applications in random matrices, integrable systems, and numerical quadrature. New results and methods are presented in the papers as well as a careful choice of open problems, which can foster interest in research in these mathematical areas. This volume also includes a brief account of the scientific contributions by Guillermo López Lagomasino.

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