CONTEMPORARY MATHEMATICS

522

Vector Bundles and Complex Geometry

Conference on Vector Bundles in Honor of S. Ramanan on the Occasion of his 70th Birthday June 16–20, 2008 Miraflores de la Sierra, Madrid, Spain

> Oscar García-Prada Peter E. Newstead Luis Álvarez-Cónsul Indranil Biswas Steven B. Bradlow Tomás L. Gómez Fditors



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Contents

Preface	ix
The Work of S. Ramanan M. S. Narasimhan	1
Parabolic Bundles on Algebraic Surfaces II - Irreducibility of the Moduli Space V. Balaji and A. Dey	7
Finite Subgroups of $\operatorname{\mathbf{\mathbf{PGL}}}_{2}(K)$ Arnaud Beauville	23
Picard Groups of Moduli Spaces of Torsionfree Sheaves on Curves USHA N. BHOSLE	31
On the Moduli of Orthogonal Bundles on a Nodal Hyperelliptic Curve USHA N. BHOSLE	43
Hilbert Schemes of Fat r-Planes and the Triviality of Chow Groups of Complete Intersections André Hirschowitz and Jaya NN Iyer	53
Vector Bundles and the Icosahedron NIGEL HITCHIN	71
Cohomology of the Toroidal Compactification of \mathcal{A}_3 KLAUS HULEK and ORSOLA TOMMASI	89
Quasi-Complete Homogeneous Contact Manifold Associated to a Cubic Form JUN-MUK HWANG and LAURENT MANIVEL	105
Maximal Weights in Kähler Geometry: Flag Manifolds and Tits Distance (with an Appendix by A. H. W. Schmitt)	
I. Mundet i Riera	113
Orthogonal Bundles Over Curves in Characteristic Two CHRISTIAN PAULY	131
The Atiyah-Singer Index Theorem M. S. RAGHUNATHAN	141
Spin(7) Instantons and the Hodge Conjecture for Certain Abelian Four-folds: A Modest Proposal T. B. BAMADAS	155
KLAUS HULEK and ORSOLA TOMMASI Quasi-Complete Homogeneous Contact Manifold Associated to a Cubic Form JUN-MUK HWANG and LAURENT MANIVEL Maximal Weights in Kähler Geometry: Flag Manifolds and Tits Distance (with an Appendix by A. H. W. Schmitt) I. MUNDET I RIERA Orthogonal Bundles Over Curves in Characteristic Two CHRISTIAN PAULY The Atiyah-Singer Index Theorem M. S. RAGHUNATHAN Spin(7) Instantons and the Hodge Conjecture for Certain Abelian Four-folds: A Modest Proposal	105 113 131

viii	CONTENTS

Remarks on Parabolic Structures C. S. Seshadri	171
Iterated Destabilizing Modifications for Vector Bundles with Connection CARLOS SIMPSON	183

Preface

This volume is dedicated to S. Ramanan on the occasion of his 70th birthday. Ramanan's influence on mathematics is the common thread running through all the articles in this volume. In some cases it is seen directly in the subject matter of the research, while in other cases it is through his association—as colleague or mentor—with the author.

Ramanan has made important contributions to Differential and Algebraic Geometry. He has been a leading expert on vector bundles and moduli spaces for over 40 years. For many years a Distinguished Professor at the Tata Institute of Fundamental Research in Mumbai, he is now an honorary professor at the Institute of Mathematical Sciences in Chennai and Adjunct Professor at the Chennai Mathematics Institute. He is a Fellow of all three Indian National Science Academies and is a recipient of the Shanti Swarup Bhatnagar Prize, the Srinivasa Ramanujan Birth centenary award and the Third World Academy of Sciences Prize for Mathematics, among other things. Some of his many profound contributions are described in the article in this volume by M.S. Narasimhan.

Most of the articles come from a workshop on vector bundles, held in the Miraflores conference center outside Madrid, Spain in June 2008. In addition to being a felicitation for Ramanan, the workshop served several purposes:

- It was the 2008 annual workshop of the Vector Bundles on Algebraic Curves (VBAC) group. Founded in 1994 by Peter Newstead, the VBAC group has been an influential champion of vector bundles and their evolving place in mathematics. Ramanan is not only an early member of the renowned 'Tata school' of vector bundles, but has been a central member of VBAC from its first days.
- The workshop was part of a semester-long program on Moduli Spaces, organized by CSIC (Madrid), Universidad del País Vasco, Universidad de Salamanca and Universitat de Barcelona. Other activities in the program included an International School on Geometry and Physics: moduli spaces in geometry, topology and physics (Cantabria, 25-29 February 2008), a workshop on Moduli spaces of vector bundles: algebro-geometric aspects (Barcelona, 12-14 March 2008), and a workshop on Bundles, gerbes and derived categories in string theory (Salamanca, 14-16 May 2008)

Finally, no description of the Miraflores workshop would be complete without a mention of the evening of classical Indian song, with performances by workshop participants Kavita Ramanan, Tomás Gómez, V. Balaji, C.S. Seshadri, and Ramanan himself. With its minimum of fuss but abundance of beauty and skill, the memorable evening was a perfect metaphor for Ramanan's contribution to mathematics.

This volume contains a collection of papers from the Conference on Vector Bundles held at Miraflores de la Sierra, Madrid, Spain on June 16–20, 2008, which honored S. Ramanan on his 70th birthday.

The main areas covered in this volume are vector bundles, parabolic bundles, abelian varieties, Hilbert schemes, contact structures, index theory, Hodge theory, and geometric invariant theory. Professor Ramanan has made important contributions in all of these areas.



