

CONTEMPORARY MATHEMATICS

539

Combinatorics and Physics

Mini-Workshop on Renormalization

December 15–16, 2006

Conference on Combinatorics and Physics

March 19–23, 2007

Max-Planck-Institut für Mathematik

Bonn, Germany

Kurusch Ebrahimi-Fard

Matilde Marcolli

Walter D. van Suijlekom

Editors



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Preface

These proceedings are based on two meetings held at the Max-Planck-Institut für Mathematik in Bonn, Germany. Both events were centered around algebraic combinatorics in theoretical physics. The short workshop *Renormalization* was held in December 2006 and aimed at bringing together leading specialists to discuss most recent progress in the understanding of mathematical structures—especially algebro-combinatorial ones—in perturbative renormalization of quantum field theory. It included talks given by D. Kreimer, E. Kraus and J. M. Gracia-Bondía. The positive resonance experienced at this meeting led to the follow-up conference *Combinatorics and Physics* held in March 2007. This one week event was broader in its aims and focused on algebraic and combinatorial aspects in modern physics. Besides a rich spectrum of research talks, it included four mini-courses given by leading researchers with, moreover, an acknowledged reputation for being gifted lecturers.

In order to put the two meetings—and this proceedings—into context, let us start with some historical remarks. In the last ten years, a convergence has taken place between seemingly separate fields in mathematics and physics: noncommutative geometry, renormalization in perturbative quantum field theory, and numerical integration of dynamical systems and flows on manifolds. It turned out that these fields share a common (co)algebraic formalism with as key words: combinatorics on trees, Hopf algebras and Lie series. For instance, in the 2007 meeting this is reflected in the talks given by A. Connes, F. Brown, A. Iserles and H. Munthe-Kaas, ranging from noncommutative geometry and multiple zeta values to Lie-Butcher theory in the context of numerical integration methods.

One of the main notions that appear in this context is the one of combinatorial Hopf algebras. Paramount in these developments was the Connes–Kreimer application of unordered rooted trees and the related combinatorial Hopf algebras to renormalization in perturbative quantum field theory, as well as the discovery of truly unexpected connections to the famous Riemann–Hilbert problem, which was eventually addressed in full detail by Connes and Marcolli. In fact, based on the seminal findings of D. Kreimer in the late 1990s, uncovering the fundamental Hopf algebra structure of Feynman rules, the path breaking work of Connes and Kreimer created a new common and fruitful playground for mathematicians and theoretical physicists alike. It reinforced interest in algebro-combinatorial aspects in renormalization theory, culminating into a whole industry of research that lies at the interface of general quantum field theory and advanced algebra and combinatorial methods. Many foundational aspects in this new field have been elaborated, raising new questions which stand at the forefront of current research.

These meetings could not have taken place without the complete financial, logistic and organisational support from the Max-Planck-Institut für Mathematik. We thank Gerd Faltings and Don Zagier for their encouragement and interest in these meetings. We are deeply indebted to Julia Löwenstein and Dagobert Jarisch of the administration of Max-Planck-Institut für Mathematik. Without their professional help and unlimited effort these meetings would not have been possible.

We thank the speakers for their valuable contributions and all participants for their enthusiasm. Together this created an exciting atmosphere resulting in two memorable events. We especially thank Pierre Cartier, Predrag Cvitanović, Philippe di Francesco and Frédéric Patras for giving interesting and stimulating mini-courses.

September 2010

Kurusch Ebrahimi-Fard
Matilde Marcolli
Walter D. van Suijlekom

List of participants

| | |
|------------------------|-------------------------|
| Christian Brouder | Sylvie Paycha |
| Francis Brown | Abhijnan Rej |
| Pierre Cartier | Walter D. van Suijlekom |
| Alain Connes | Christophe Tollu |
| Predrag Cvitanović | Xavier Viennot |
| G rard H.E. Duchamp | Fabien Vignes-Tourneret |
| Kurusch Ebrahimi-Fard | Raimar Wulkenhaar |
| Lo c Foissy | Don Zagier |
| Alessandra Frabetti | Paul Zinn-Justin |
| Philippe Di Francesco | |
| Roland Friedrich | |
| Hanno Gottschalk | |
| Jos  M. Gracia-Bond a | |
| Massimiliano Gubinelli | |
| Ralf Holtkamp | |
| Arieh Iserles | |
| Thomas Krajewski | |
| Elisabeth Kraus | |
| Dirk Kreimer | |
| Jean-Louis Loday | |
| Dominique Manchon | |
| Matilde Marcolli | |
|  ngela Mestre | |
| Hans Munthe-Kaas | |
| Daniele Oriti | |
| Fr d ric Patras | |

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This book is based on the mini-workshop Renormalization, held in December 2006, and the conference Combinatorics and Physics, held in March 2007. Both meetings took place at the Max-Planck-Institut für Mathematik in Bonn, Germany.

Research papers in the volume provide an overview of applications of combinatorics to various problems, such as applications to Hopf algebras, techniques to renormalization problems in quantum field theory, as well as combinatorial problems appearing in the context of the numerical integration of dynamical systems, in noncommutative geometry and in quantum gravity.

In addition, it contains several introductory notes on renormalization Hopf algebras, Wilsonian renormalization and motives.

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