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Interactions Between Hyperbolic Geometry, Quantum Topology and Number Theory

Workshop
June 3–13, 2009
Conference
June 15–19, 2009
Columbia University, New York, NY

Abhijit Champanerkar, Oliver Dasbach,
Efstratia Kalfagianni, Ilya Kofman,
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Dedicated to the memory of Xiao-Song Lin,
whose mathematics continues to inspire.

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Preface

This book is based on a 10-day workshop of minicourses in hyperbolic geometry, quantum topology and number theory, given by leading experts, in June 2009 at Columbia University. The proceedings of this enormously successful workshop can serve as an introduction to this active research area in a way that is expository and broadly accessible to graduate students and researchers new to this area.

The twelve papers in this book present ideas and tackle problems that arose from the confluence of several different currents of research mathematics over the past thirty years. First, Thurston's ground-breaking work established the importance of hyperbolic geometry in the study of 3-manifolds. Mostow-Prasad rigidity implied that geometric invariants are also topological invariants, providing new tools from geometry to study 3-manifolds. Second, rigidity also enabled the use of arithmetic tools to study hyperbolic 3-manifolds giving rise to arithmetic topological invariants. Third, following the discovery and subsequent generalizations of the Jones polynomial, new quantum diagrammatic invariants of 3-manifolds were discovered. How these geometric, arithmetic and quantum invariants interact is a fundamental open question in the study of knots, links and 3-manifolds that motivates much of the research discussed in this book.

Quantum invariants such as the Jones polynomial are not understood in terms of the geometry of the complementary space. But a recurrent theme is that geometric and quantum invariants can be related by quantization of geometric invariants. Kashaev, Garoufalidis, Baseilhac and Dimofte & Gukov each discuss aspects of geometric quantization. The big open problem in this area is the Volume Conjecture, which relates hyperbolic volume to Jones polynomials. Murakami, Dimofte & Gukov, and Kashaev each discuss different approaches to the Volume Conjecture.

A basic tool in the study of hyperbolic 3-manifolds is an ideal triangulation, which can be described in terms of complex shape parameters that satisfy certain polynomial gluing equations. These equations are in general difficult to solve. Futer & Guéritaud and Luo discuss angle structures, which provide an alternative method to solve the gluing equations and understand the geometry of the 3-manifold. Purcell discusses the geometry of augmented link complements, which can be explicitly described without having to solve gluing equations.

Rigidity implies that the shape parameters are algebraic numbers, which leads to arithmetic invariants to study hyperbolic 3-manifolds. The resulting arithmetic tools have been used to study geodesic surfaces, closed geodesics and volumes of hyperbolic 3-manifolds. Neumann discusses arithmetic invariants like the invariant trace field and Bloch invariants, and realization problems related to these invariants. Deforming hyperbolic structures on 3-manifolds gives rise to algebraic curves defined over number fields which are discussed by Long & Reid. Walsh discusses

applications of arithmetic invariants to study the problem of commensurability of hyperbolic knots and 3-manifolds. Garoufalidis discusses how arithmetic invariants arise in the study of the asymptotic behavior of quantum invariants.

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This book is based on a 10-day workshop given by leading experts in hyperbolic geometry, quantum topology and number theory, in June 2009 at Columbia University. Each speaker gave a minicourse consisting of three or four lectures aimed at graduate students and recent PhDs. The proceedings of this enormously successful workshop can serve as an introduction to this active research area in a way that is expository and broadly accessible to graduate students.

Although many ideas overlap, the twelve expository/research papers in this volume can be grouped into four rough categories:

(1) different approaches to the Volume Conjecture, and relations between the main quantum and geometric invariants;

(2) the geometry associated to triangulations of hyperbolic 3-manifolds;

(3) arithmetic invariants of hyperbolic 3-manifolds;

(4) quantum invariants associated to knots and hyperbolic 3-manifolds.

The workshop, the conference that followed, and these proceedings continue a long tradition in quantum and geometric topology of bringing together ideas from diverse areas of mathematics and physics, and highlights the importance of collaborative research in tackling big problems that require expertise in disparate disciplines.

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