

# CONTEMPORARY MATHEMATICS

542

## Harmonic Maps and Differential Geometry

A Harmonic Map Fest in Honour of  
John C. Wood's 60th Birthday  
September 7–10, 2009  
Cagliari, Italy

E. Loubeau  
S. Montaldo  
Editors



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## Preface

This volume is the proceedings of a conference held in Cagliari, Italy, in September 2009.

The meeting was organized to celebrate Professor J. C. Wood's contributions to harmonic maps on the occasion of his sixtieth birthday and brought together many of his former students, collaborators and friends.

The broad spectrum covered by the articles will reflect the numerous research interests and activities of Prof. Wood, and a retrospective look at his career can be read in the introductory piece "Thirty-nine years of harmonic maps".

We also intend to offer here a picture of some of the more recent developments in the theory of harmonic maps as well as several, closely or loosely, related topics.

Saluted by a number of speakers and mentioned, through his work, by most, the ever-present figure of James Eells presided over the assembly.

It is our pleasant duty to thank the Scientific Committee of the conference: P. Baird, F. Burstall, L. Lemaire and S. Salamon. Their continual advice was key to the success of the meeting.

We also thank the members of the Organizing Committee: S. Gudmundsson and R. Pantilie whose hard work contributed to plan the meeting.

Special thanks go to the Local Organizing Committee: G. Bande, R. Caddeo, G. Franzoni, M. Musio and P. Piu, who were crucial in fixing local problems and creating the right atmosphere.

Thanks are also due to all participants and in particular to speakers.

The congress would not have been possible without funding by the Università di Cagliari — Dipartimento di Matematica e Informatica, GNSAGA-INdAM, Prin-2007: Riemannian metrics and differentiable manifolds, Fondazione Banco di Sardegna, Regione Autonoma della Sardegna, Provincia di Cagliari and Cantina Sociale di Santadi.

We would also like to thank the AMS for publishing this volume and C. Thivierge for her help.

E. Loubeau and S. Montaldo

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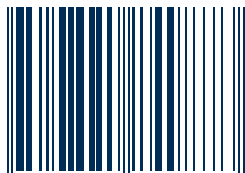


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This volume contains the proceedings of a conference held in Cagliari, Italy, from September 7–10, 2009, to celebrate John C. Wood's 60th birthday.

These papers reflect the many facets of the theory of harmonic maps and its links and connections with other topics in Differential and Riemannian Geometry. Two long reports, one on constant mean curvature surfaces by F. Pedit and the other on the construction of harmonic maps by J. C. Wood, open the proceedings. These are followed by a mix of surveys on Prof. Wood's area of expertise: Lagrangian surfaces, biharmonic maps, locally conformally Kähler manifolds and the DDVV conjecture, as well as several research papers on harmonic maps. Other research papers in the volume are devoted to Willmore surfaces, Goldstein-Pedrich flows, contact pairs, prescribed Ricci curvature, conformal fibrations, the Fadeev-Hopf model, the Compact Support Principle and the curvature of surfaces.

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