

CONTEMPORARY MATHEMATICS

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New Developments in Lie Theory and Its Applications

Seventh Workshop on
Lie Theory and Its Applications
November 27–December 1, 2009
Córdoba, Argentina

Carina Boyallian
Esther Galina
Linda Saal
Editors



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Preface

This book represents the proceedings of the *Seventh Workshop in Lie Theory and Its Applications* that was held in Córdoba, at the Facultad de Matemática Astronomía y Física, Universidad Nacional de Córdoba, Argentina, from November 27 to December 1st, 2009. It was preceded by a special event, *Encuentro de Teoría de Lie*, held November 23 to 26, in honor of the sixtieth birthday of Jorge A. Vargas, who contributed strongly to the development of Lie theory in Córdoba.

At the very beginning, the series of workshops focused on the theory of Lie groups and their representations, and later, they included as central topics several applications of Lie theory in general.

The main focus in these proceedings are representation theory, harmonic analysis on Lie groups and mathematical physics related with Lie theory.

The invited contributors provide a broad overview on these subjects and also on the recent developments of their research. The aim of this volume is to bring to a greater audience not only the contents of the articles but also the bridging of ideas experienced during the workshop.

In algebraic combinatorics, much research has been done in distance regular graphs. F. Levstein, C. Maldonado and D. Penazzi consider the set of functions $L^2(X)$, where X is the set of vertices of the dual polar graph and study an adjacency operator on $L^2(X)$ given rise by the distance, and its decomposition into eigenspaces. They associate a lattice to the graph, map the lattice onto $L_2(X)$ and study the interaction between this map and the adjacency operator, which allows them to characterize the eigenspaces of this operators in terms of this lattice. Instead of an orthogonal basis they can give a tight frame for each eigenspace. They give a formula for the constants associated to each tight frame, and in the case of the eigenspace corresponding to the second largest eigenvalue of the adjacency operator they compute the constant in a more closed form. As an application they answer a problem posed by Paul Terwilliger: a formula for the product in the Norton algebra attached to the eigenspace corresponding to the second largest eigenvalue of the adjacency operator. This is a nice article accessible even for non-experts in algebraic combinatorics.

Jacques Faraut presents a comprehensive exposition on some results on asymptotic harmonic analysis related to inductive limits of increasing sequences of Gelfand pairs and the corresponding spherical functions. In this article he considered three cases: multivariate Bessel functions associated to the space of Hermitian matrices, characters of the unitary group and multivariate Laguerre polynomials associated to the Heissenberg group.

The study of restriction of square integrable representations of a semisimple Lie group has been the object of many recent developments. Jorge Vargas analyzes, for

a square integrable representation of a semisimple Lie group, the continuous spectrum of its restriction to a semisimple subgroup, and also provides explicit examples of representations so that its restriction to some particular reductive subgroup have non empty discrete spectrum.

Susanna Dann and Gestur Ólafsson give an interesting review of Paley-Wiener theorems in different contexts: Euclidean spaces, symmetric spaces of compact and noncompact type, and inductive limits of symmetric spaces. For the Euclidean case, a different version of the classical theorem is given, in terms of Fourier analysis on the Euclidean motion group. The nontrivial proof is included.

Gestur Ólafsson and Joseph A. Wolf for applications to harmonic analysis, state results on the spherical transform, on the Paley-Wiener theorem and on solutions of invariant differential operators on symmetric spaces. Furthermore, they apply these results to problems in Fourier analysis on projective/inductive limits of symmetric spaces.

Anthony H. Dooley presents a survey along with some new results on contractions associated to rank one semisimple Lie groups. In this paper the explicit formula of the Cayley transform is given, some known results on Sobolev spaces associated to the compact and non compact pictures of the principal and complementary series are described, and the uniformly boundedness of matrix entries of these series are analyzed.

The problem of understanding the metaplectic or Segal-Shale-Weil representation, has a long history with many people contributing. Luis Gutiérrez, José Pantoja, and Jorge Soto-Andrade construct generalized Weil representations for involutive analogues of classical $SL(2, k)$, k a field, via generators and relations. The construction of the representation is based on a kind of Bruhat presentation of the group, already studied by Pantoja and Soto-Andrade.

The classification of finite-dimensional pointed Hopf algebras with abelian coradical by the Lifting Method of Andruskiewitsch and Schneider requires in its first step the classification of finite-dimensional Nichols algebras of diagonal type and their presentation by generators and relations. The first part of this problem was solved by Heckenberger and the second was answered in principle by Angiono. In the classification a bunch of examples appear which have the same root system as a contragredient Lie superalgebra. Nicolás Andruskiewitsch, Iván Angiono, and Hiroyuki Yamane address the problem of finding an explanation of this phenomenon. They work out a correspondence between super structures and their bosonizations. As their main result, they identify Nichols algebras of diagonal type which have a root system of a contragredient Lie superalgebra. Further, they give explicit lists of generators and relations. This paper is an important part of the classification of pointed Hopf algebras and clarifies the role of quantized contragredient Lie superalgebras in this classification.

Quasireductive Lie superalgebras, namely, Lie superalgebras with reductive even part and semisimple (as a module over the even part) odd part have a manageable structure and representation theory. Vera Serganova, in a nice article, shows this, giving a decomposition of the quasireductive Lie superalgebra into a semi-direct sum of some ideal and a Lie subalgebra and an explicit descriptions of such parts. A description of the space of superfunctions on a quasireductive Lie supergroup as a left module over its Lie superalgebra is also given. This observation is very important and definitely can be used to study the space of superfunctions

on a homogeneous superspace. Also it is shown that for a simple module over a quasireductive Lie superalgebra there exists a highest weight and that two such modules are isomorphic (up to parity) if and only if they have the same highest weight.

The workshop would not have been possible without the generous support of International Center for Theoretical Physics (Italy), Centro de Investigaciones y Estudios en Matemática de Córdoba, Consejo Nacional de Investigaciones Científicas y Tecnológicas, Agencia Córdoba Ciencia, Facultad de Matemática, Astronomía y Física, Secretaria de Ciencia y Tecnología (Universidad Nacional de Córdoba) and International Mathematical Union.

Córdoba, February 2011

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This volume focuses on representation theory, harmonic analysis in Lie groups, and mathematical physics related to Lie theory. The papers give a broad overview of these subjects and also of the recent developments of the authors’ research.

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