

CONTEMPORARY MATHEMATICS

545

Concentration, Functional Inequalities and Isoperimetry

International Workshop on
Concentration, Functional Inequalities and Isoperimetry
October 29–November 1, 2009
Florida Atlantic University
Boca Raton, Florida

Christian Houdré
Michel Ledoux
Emanuel Milman
Mario Milman
Editors



American Mathematical Society

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Preface

The interconnections between Concentration, Isoperimetry and Functional Inequalities originated several decades ago in the study of Sobolev inequalities, Banach Spaces and Convexity, Gaussian Measures and Markov Processes. It has since been progressively realized that the study of these problems requires taking into account certain important geometric structures associated with the underlying metric and measure (or distribution law) in high dimensional and infinite dimensional spaces. It was also understood that the crux of these problems is related to rather general probabilistic phenomena. Similar considerations turned out to be very useful and powerful in other subfields of Mathematics and parallel developments occurred in some other areas such as Asymptotic Geometric Analysis, Convex Geometry, Probability in Banach Spaces and the “local theory” thereof, Combinatorics, Harmonic Analysis, Random Matrices and Stochastic Processes. Moreover, these methods were found to have numerous important applications in these areas as well as in Statistics and Computer Science. This breadth is very well illustrated by the contributions present in this volume.

Most of the papers in this volume were presented at the International Conference held at Florida Atlantic University in Boca Raton on October 29–November 1, 2009. The organizers of this conference are grateful for the support provided by the NSF. Many thanks are due to Emily Cimillo for her help in coordinating and running the conference and to Annette Rohrs for her great editorial work.

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The volume contains the proceedings of the international workshop on Concentration, Functional Inequalities and Isoperimetry, held at Florida Atlantic University in Boca Raton, Florida, from October 29–November 1, 2009.

The interactions between concentration, isoperimetry and functional inequalities have led to many significant advances in functional analysis and probability theory. Important progress has also taken place in combinatorics, geometry, harmonic analysis and mathematical physics, to name but a few fields, with recent new applications in random matrices and information theory.

This book should appeal to graduate students and researchers interested in the fascinating interplay between analysis, probability, and geometry.

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