

CONTEMPORARY MATHEMATICS

549

Symmetries and Related Topics in Differential and Difference Equations

Jairo Charris Seminar 2009
Symmetries of Differential and Difference Equations
Escuela de Matemáticas
Universidad Sergio Arboleda
Bogotá, Colombia

David Blázquez-Sanz
Juan J. Morales-Ruiz
Jesús Rodríguez Lombardero
Editors



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Jairo Charris. The mathematician. The seminar



Jairo Antonio Charris-Castañeda was one of the most relevant Colombian mathematicians in the last third of the past century. He was born in Ciénaga, Magdalena, Colombia, on November 21, 1939. During his childhood he was a baseball player, until he discovered that he enjoyed scientific activities and classical music more than baseball.

He obtained a degree in chemical engineering in 1962 and in mathematics in 1967, both from Universidad Nacional de Colombia. In 1969 he received a master's degree in mathematics from the University of Chicago under the supervision of Professor R. Narasimhan. From 1981 to 1984, under the supervision of Professor M. E. H. Ismail, he worked in the department of mathematics of Arizona State University on Pollaczek orthogonal polynomials, writing his Ph.D. thesis on this subject. He came back to Colombia and continued his scientific career as a professor at Universidad Nacional de Colombia, Bogotá, until his retirement in 1998. After his retirement, he worked at Universidad Nacional and Universidad Sergio Arboleda, both in teaching and research activities.

The scientific fields where Professor Charris focused his research were the theory of orthogonal polynomials (in particular, his contributions on sieved polynomials had a strong impact on the mathematical community), complex analysis, compactification theory, and group theory. He authored 32 papers and three monographs. His scientific leadership was quite remarkable: he supervised 20 graduate master's and doctoral students from 1976 to 2001.

Professor Charris died on July 17, 2003, as a consequence of a serious illness. Four years after his death the *Jairo Charris Seminar* was established in his memory

as an annual meeting at the Universidad Sergio Arboleda *Instituto de Matemáticas y sus Aplicaciones* (IMA). The first seminar took place in 2007. Each year the theme of the seminar changes. Here is the list of the previous Seminars:

- 2007. Differential Algebra. Bogotá, Colombia.
- 2008. Complex Analysis and Orthogonal Polynomials. Bogotá, Colombia.
- 2009. Symmetries of Differential and Difference Equations. Bogotá, Colombia.
- 2010. Algebraic Aspects of Darboux Transformations, Quantum Integrable Systems and Supersymmetric Quantum Mechanics. Santa Marta, Colombia.

Due to the high quality of the speakers and talks presented at the Jairo Charris Seminar, the IMA, in cooperation with the American Mathematical Society, has been publishing the Proceedings of these meetings as part of the Contemporary Mathematics book series. The first volume published as part of this cooperation agreement was *Differential Algebra, Complex Analysis and Orthogonal Polynomials* (P. Acosta-Humánez and F. Marcellán, eds.), Contemp. Math., vol. **509**, Amer. Math. Soc., Providence, RI, 2010.

The IMA expresses its gratitude to the contributors and participants who made each seminar a memorable event. We wish to thank Christine Thivierge of the AMS for her efficient support in the publication of these proceedings.

Finally, we would like to thank Jairo Charris' friends Víctor Albis, Jaime Lesmes, and Francisco Marcellán for their helpful comments and suggestions in writing this foreword and express our gratitude to Professor Charris' family for the opportunity to develop this beautiful project.

Primitivo B. Acosta-Humánez
Director IMA (Instituto de Matemáticas y sus Aplicaciones)

Preface

This volume represents the 3rd Jairo Charris seminar entitled “Symmetries of differential and difference equations”, which was held at Universidad Sergio Arboleda, in Bogotá, Colombia in August 2009. The aim of this conference was to discuss recent developments and several approaches to the geometrical and algebraic aspects of differential and difference equations, such as Lie symmetry groups and their invariants, differential Galois theory, group theoretical methods in physics, and geometrization of mechanics.

The contributions by Ibragimov, Jiménez, and Olver relate to Lie symmetries, equivalence transformations, and differential invariants. The paper by Ibragimov is a survey on integration methods for parabolic equations. The equivalence problem for parabolic equations is considered, and the equations that can be reduced to the heat equation by certain equivalence transformation are characterized by terms of a differential semi-invariant. Also some classical formulas for closed form solutions of heat equation are revisited. Jiménez’s contribution gives some applications of the theory of Lie correspondences, a geometrical dictionary that translates systems of partial differential equations of different orders and number of variables. This theory was proposed by S. Lie at the end of the 19th century but was only recently developed. As an example, the theory of characteristics can be viewed as a simple application of Lie correspondences. Olver’s paper surveys the topic of differential invariants, focusing on explicit computations based on moving frames and Gröbner bases methods. The cases of finite dimensional Lie groups and infinite dimensional pseudogroups are analyzed, and most recent results on the topic are surveyed.

The papers by Aparicio Monforte and Weil, Mozo, and Sauloy relate to group theoretical methods in linear equations, namely differential Galois theory and Stokes phenomenon. The work by Aparicio and Weil is devoted to the study of the structure of the Galois groups of higher order variational equations as an essential tool for understanding the integrability of Hamiltonian systems. In particular, they propose effective tools for proving the nonintegrability of Hamiltonian systems by means of Morales–Ramis approaches. As a concrete example to test their method they give a new systematic proof of the nonintegrability of the degenerated case of the Hénon–Heiles family. The paper by Mozo surveys the theory and a number of applications of the summability of solutions of linear differential equations and some related problems in complex dynamics. Sauloy introduces the Stokes phenomenon for linear q -difference equations and its connection with q -difference Galois group. As an interesting illustration he analyzes the q -Euler equation and Tshakaloff series.

The works by Muñoz and Wolf examine the development of some geometrical methods in theoretical physics. The paper by Muñoz deals with two fundamental problems of theoretical mechanics: the mathematical nature of time in Lagrangian mechanics and the preservation of the equations of motion by changes of frame. New aspects of those problems are unveiled in his approach. The paper by Wolf is devoted to discrete Hamiltonian systems arising in geometrical optics and analogous to those of finite quantum mechanics.

The editors would like to acknowledge the assistance of the Instituto de Matemáticas y sus Aplicaciones at Universidad Sergio Arboleda as host institution for the third Jairo Charris Seminar. The valuable work of Primitivo Acosta-Humánez and Reinaldo Nuñez made possible the success and continuity of this meeting.

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The papers include topics such as Lie symmetries, equivalence transformations and differential invariants, group theoretical methods in linear equations, namely differential Galois theory and Stokes phenomenon, and the development of some geometrical methods in theoretical physics.

The reader will find new interesting results in symmetries of differential and difference equations, applications in classical and quantum mechanics, two fundamental problems of theoretical mechanics, the mathematical nature of time in Lagrangian mechanics and the preservation of the equations of motion by changes of frame, and discrete Hamiltonian systems arising in geometrical optics and analogous to those of finite quantum mechanics.

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